Homogenization of a triply-nonlinear parabolic-hyperbolic system

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Maxwell's equations in a nonhomogeneous and anisotropic metal surrounded by an insulating enviroment lead to a parabolic-hyperbolic system of partial differential equations in the whole space. We provide a weak formulation for an initial-value problem with nonlinear constitutive relations of the form

$$\vec{B} = \vec{\mathcal{F}}(\vec{H}, x), \qquad \vec{J} = \vec{\mathcal{G}}(\vec{E}, \vec{H}, x).$$

We allow for possible discontinuities of $\vec{\mathcal{F}}(\cdot, x)$, thus accounting for the possible occurrence of free boundaries, but we neglect hysteresis. We prove the existence of a solution. We then assume that the above constitutive relations exhibit fast periodic oscillations in space, let the space-period vanish, and prove two-scale convergence (in the sense of Nguetseng) to a two-scale homogenized problem.

A simpler analogous problem is issued from phase transitions. Open questions include the possible uniqueness of the solution.