Global solution to an abstract doubly nonlinear Volterra equation

Gianni Gilardi, Pavia

Some thermodynamically consistent mathematical models for phase transition accounting for memory effects involve equations like

$$\partial_t \beta(u) + \operatorname{div} \mathbf{Q} = f \qquad \text{where} \qquad \mathbf{Q} = -\big(\alpha(\nabla u) + k * \alpha(\nabla u)\big) \tag{1}$$

for the absolute temperature u. In (1), β is a real monotone function defined in some interval (and the choice $\beta = \ln$, the logarithm, looks particularly appropriate), $\alpha : \mathbb{R}^n \to \mathbb{R}^n$ is monotone, and $k : (0,T) \to \mathbb{R}$ is a memory kernel. Several initial-boundary value problems for (1) in bounded domains are particular cases of the abstract Cauchy problem

$$(B(u))' + A(u) + k * A(u) \ni f \quad \text{in } (0,T) \qquad \text{and} \qquad B(u)|_{t=0} \ni v^0 \tag{2}$$

where $A : V \to V^*$ and $B : H \to H$ are maximal monotone operators in Banach spaces, namely, V is a reflexive Banach space and H is a Hilbert space such that $V \subset H \subset V^*$ with compact embeddings. In a joint work with Ulisse Stefanelli (Pavia), an existence result for (2) has been proved for coercive and bounded operators A and (possibly degenerate and singular) subdifferentials B, under a (widely satisfied) compatibility condition, with k very general. The present talk deals with the outline of such a result.