Elastic materials with a stretching threshold

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Some biological tissues (tendons, ligaments) exhibit a sharp reduction of deformability beyond some stress threshold, below which they may be considered elastic. This behaviour can be effectively extrapolated to a limit situation in which the material becomes undeformable beyond some deformation threshold (to which a stress threshold can be associated). In the fully stretched material the stress can take any value above threshold. As a model problem we have considered the motion of a layer of such a material in which one boundary is kept fixed while to the other a tangential stress beyond threshold is applied. The corresponding mathematical model is formulated as a hyperbolic free boundary problem in which at each time instant the interface is made of the points reaching the threshold stretching. It is shown that two cases are possible: (i) the stress is continuous at the interface, (ii) the stress has an unknown jump across the interface. It is proved that the velocity field has the same behaviour and that in case (i) the interface velocity is always lower than the speed of sound in the material, while in case (ii) the interface must be a characteristic of the governing equation in the elastic phase. An existence and uniqueness theorem is proved together with some qualitative properties of the solution.

This is a joint work with Angiolo Farina, Lorenzo Fusi, and Kumbakonam R. Rajagopal.