Numerical analysis of a convolution model of phase separation

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The integro-differential equation (IDE)

$$u_t \;=\; \varepsilon \int_\Omega J(x-y)(u(y,t)-u(x,t))\,dy + f(u)$$

with given kernel $J \geq 0$ and bistable $f(u)$ (e. g., $f(u) = u - u^3$), has been proposed as a model of phase separation. It is a non-local analogue of the more usual Allen-Cahn equation

$$u_t \;=\; \varepsilon \Delta u + f(u),$$

but its behaviour and numerical analysis are different in important aspects. This talk concentrates on the numerical approximation of this IDE, and highlights properties that can be exploited to obtain results surprisingly efficiently.