Newton Iteration Procedure and Nonlinear Elliptic Boundary Value Problems with Non-Smooth Data

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The lecture concerns boundary value problems for quasilinear second order elliptic equations and systems with non-smooth data. Here *non-smooth data* means that the domain can be non-smooth, that the boundary conditions can change type (mixed boundary conditions, where the Dirichlet and the Neumann boundary parts can touch) and that the coefficients of the equations and the boundary conditions can be discontinuous with respect to the space variable x (but they have to be smooth with respect to the unknown function u and its gradient ∇u). The equations are of divergence type (this is joint work with K. Gröger) as well as of non-divergence type (joint work with D. Palagachev and L. Softova).

The aim is to state conditions which imply results of the following type: Let u_0 be a weak solution such that the formally linearized (in $u = u_0$) boundary value problem has no nontrivial solution. Let u_1 be sufficiently close to u_0 in $L^\infty(\Omega) \cap W^{1,2}(\Omega)$ (in the case of divergence type equations) or in $W^{1,\infty}(\Omega)$ (in the case of non-divergence type equations), respectively, and let $u_2, u_3, \ldots$ be the Newton iterations determined by means of the formally linearized (in $u = u_1$, $u_2, \ldots$) boundary value problem. Then $u_l \to u_0$ in $L^\infty(\Omega) \cap W^{1,2}(\Omega)$ (in the case of divergence type equations) or in $W^{1,\infty}(\Omega)$ (in the case of non-divergence type equations), respectively.