Regularity of Displacement and Stresses in Linear and Nonlinear Elasticity with Mixed Boundary Conditions

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We prove $W^{1,p}$ regularity for the displacements $\boldsymbol{u}$ and L^p regularity for the stresses $\boldsymbol{\sigma}$ for systems of nonlinear elasticity with nonsmooth coefficients and mixed boundary conditions for some $p > 2$. In particular, we consider the system

$$\begin{aligned} -\operatorname{div} \boldsymbol{b}(x, \boldsymbol{\varepsilon}(\boldsymbol{u})(x)) &= \boldsymbol{f}(x) \text{ in } \Omega \\ \boldsymbol{n}(x) \cdot \boldsymbol{b}(x, \boldsymbol{\varepsilon}(\boldsymbol{u})(x)) &= \boldsymbol{g}(x) \text{ on } \Gamma_N \\ \boldsymbol{u} &= 0 \text{ on } \Gamma_D = \Gamma \setminus \Gamma_N, \end{aligned}$$

where $\boldsymbol{b} : \Omega \times \mathbb{R}^{n\times n}_{\mathrm{sym}} \to \mathbb{R}^{n\times n}_{\mathrm{sym}}$ is the nonlinear stress strain relation. The bounded domain Ω is assumed to be regular in the sense of Gröger and to have a Lipschitz boundary. The specific maximum value of p depends only on the domain, the regularity of the applied loads, and the coercivity and boundedness constants of the nonlinear operator $\boldsymbol{b}$. This result is of importance for instance for the discussion of elasticity systems with temperature dependent Lamé coefficients. Given that the temperature varies between certain bounds, our result implies the existence of a unique solution in $W^{1,p}$ with p independent of the actual temperature field.