Thin-film limits of functionals on $\mathcal{A}$-free vector fields

Carolin Kreisbeck

WIAS Berlin, Universität Regensburg

Working with variational principles subject to linear PDE constraints conveyed by a constant-rank operator $\mathcal{A}$ allows us to treat a number of problems in continuum mechanics and electromagnetism in a unified way. The topic of this talk, which reports on a joint work with Filip Rindler (University of Warwick), is 3d-2d dimension reduction within this general framework. We study the effective behavior of integral functionals as the thickness ε of the domain tends to zero. Under certain conditions we show that the Γ-limit is an integral functional and give an explicit formula. The limit functional turns out to be constrained to $\mathcal{A}_0$-free vector fields, where the limit operator $\mathcal{A}_0$ is in general not of constant rank. While the lower bound follows from a Young measure approach together with a new decomposition lemma, the construction of a recovery sequence relies on algebraic considerations in Fourier space. This part of the argument requires a careful analysis of the limiting behavior of the rescaled operators $\mathcal{A}_\varepsilon$ by a suitable convergence of their symbols, as well as an explicit construction for plane waves. As applications, we characterize a thin-film Γ-limit in micromagnetics and recover the energy of a membrane model with bending moment in nonlinear elasticity.