Cross-diffusion systems with entropy structure

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Cross-diffusion systems describe the diffusive interaction of multi-species systems. Examples include multi-species population dynamics, cell biology, ion transport through membranes, and even applications in economics. The models consist of strongly coupled parabolic equations involving a non-diagonal diffusion matrix. Generally, this matrix is neither symmetric nor positive definite such that standard techniques cannot be applied. In this talk, the recent progress on cross-diffusion systems is presented. The key idea is the observation that many systems possess a formal gradient-flow structure often originating from a thermodynamic modeling. Mathematically, this provides a convex Lyapunov functional (called an entropy), gradient estimates coming from the entropy production, and sometimes even L^∞ bounds (without using a maximum principle). The boundedness-by-entropy method is introduced, which gives the global existence of bounded weak solutions for a large class of cross-diffusion systems. We also mention results on the large-time asymptotics of solutions, the uniqueness of weak solutions, and the existence and weak-strong uniqueness of renormalized solutions, whose proofs are all based on entropy methods.