Pontryagin's Maximum Principle for SPDEs and Its Relation to Dynamic Programming

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In this talk, we consider the following optimal control problem of stochastic partial differential equations (SPDEs). Minimize

$$J(u) := \mathbb{E}\left[\int_0^T \int_\Lambda l(x_t^u(\lambda), u_t)\mathrm{d}\lambda\mathrm{d}t + \int_\Lambda h(x_T^u(\lambda))\mathrm{d}\lambda\right]$$

subject to

$$\begin{cases} \mathrm{d}x_t^u = [\Delta x_t^u + b(x_t^u, u_t)]\mathrm{d}t + \sigma(x_t^u, u_t)\mathrm{d}W_t, \quad t \in [0,T], \\ x_0^u = x_0 \in L^2(\Lambda). \end{cases}$$

In order to derive a necessary optimality condition, we employ the classical spike variation method. In addition to the first order adjoint state, in the stochastic case, one has to introduce a second order adjoint state. We give a novel characterization of this second order adjoint state as the solution to a backward SPDE on the space $L^2(\Lambda) \otimes L^2(\Lambda) \cong L^2(\Lambda^2)$. Using this representation, we prove the maximum principle for controlled SPDEs.

As another application of our characterization of the second order adjoint state, we discuss the connection between the stochastic Hamiltonian system and the associated infinite-dimensional Hamilton-Jacobi-Bellman equation in the framework of viscosity solutions.

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