Reappraisal of 2+ myths in the theory of nonlinear waves

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Abstract

The first part of the talk will consider the classic problem of bifurcation of dark solitary waves, modelled by NLS $\rightarrow$ KdV. It is shown that the resulting KdV takes a universal form

$$2\mathcal{A}'(k)q_T + \mathcal{B}''(k)qq_X + \mathscr{K}\, q_{XXX} = 0\,,$$

where q is a perturbation wavenumber, $\mathcal{A}(k)$ is the wave action, $\mathcal{B}''(k)$ is the wave action flux, and $\mathscr{K}$ is the Krein index of the periodic state at infinity. At first glance this looks like – yet another – example of the universality of the KdV equation. However, it will be shown that the KdV equation for water waves in shallow water also has the same geometric form, with wave action conservation replaced by mass conservation and $\mathscr{K}$ replaced by the Krein index of a uniform flow. Indeed, there is a universal mechanism and form for the appearance of the KdV equation [3].

The geometry of modulation is the backbone of the above KdV emergence. Modulation becomes even more interesting in higher dimension. A new mechanism for bifurcation of rolls in pattern formation to planforms is presented. A modulation equation is derived which has a sequence of multi-pulse planforms on periodic background,

$$q_T = \left(\tfrac{1}{2}\mathscr{A}_{kk}q^2 + \mathscr{K}\, q_{XX}\right)_{XX} + \mathscr{B}_\ell q_{YY}\,.$$

An example where this phenomenon occurs is the Swift-Hohenberg equation. If there is time the "+" part will present the recent result that the shallow water equations (SWEs) are not valid in the limit of shallow water [1]! The SWEs generate horizontal vorticity which is invisible until the SWEs are embedded in the Euler equations, causing breakdown of the SWEs

References:

1. TJB & D.J. Needham. *Breakdown of the shallow water equations due to growth of the horizontal vorticity*, J. Fluid Mech. **679**, 655-666 (2011).
2. TJB. *Emergence of unsteady dark solitary waves from coalescing spatially periodic patterns*, Proc. Royal Soc. London A **468** 378 4–3803 (2012).
3. TJB. *A universal form for the emergence of the KdV equation*, Proc. Royal Soc. London A (in press, 2013).