Fast diffusion equations, tails and convergence rates

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Understanding the intermediate asymptotic and computing convergence rates towards equilibria are among the major problems in the study of parabolic equations. Convergence rates depend on the tail behaviour of solutions. This observation raised the following question: how can we understand the tail behaviour of solutions from the tail behaviour of the initial datum?

In this talk, I will discuss the asymptotic behaviour of solutions to the fast diffusion equation. It is well known that non-negative solutions behave for large times as the Barenblatt (or fundamental) solution, which has an explicit expression. In this setting, I will introduce the Global Harnack Principle (GHP), precise global pointwise upper and lower estimates of non-negative solutions in terms of the Barenblatt profile. I will characterize the maximal (hence optimal) class of initial data such that the GHP holds by means of an integral tail condition. As a consequence, I will provide rates of convergence towards the Barenblatt profile in entropy and in stronger norms such as the uniform relative error.