State of the Art, Trends, and Directions in Smart Systems[1]

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With the contribution of
The Smart System Network[2]

Abstract

The state of the art presented here has been established by the participants of this European "Smart systems" network. It reflects their major examination of the three fundamental areas of modeling, optimization and control, and numerical simulation. The investigation of the modeling consists in studing the appropriate models for linear and nonlinear coupling effects, for new composite structures (with their nonlinearities induced by hysteresis, phase transitions, contact or presence of cracks) and the associated time-dependent equations. The exploration in Optimization and Control concern theoretical questions on exact controllability, stabilization, active, passive, hybrid vibration control, optimal shape design (such as the best location of a pattern of sensors, actuators, techniques for nondestructive damage detection, etc.) and practical questions about the robustness and accuracy of the design control systems. Regarding the numerical simulation, two kinds of research have to be brought to a successful conclusion at the same time: at a conceptual level dedicated to theoretical questions such as sensitivity of finite elements (locking, spill-over) and at a practical level dedicated to solve "real-life" applications (such as the simulation of a complete device in order to assess the models retained by checking whether predictions agree with experiments).

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Introduction

During the last decade, the fast development of both material sciences and technology (from design to production) has induced a growing mathematical interest in the multi-disciplinary field of smart structures. The contribution of this report is to draw a precise state of the art of the latest progresses in the modeling, control, optimization and numerical simulation of new materials and their applications to adaptive structures.

"New materials" is used here as a generic term for "functional" or "multifunctional" materials (such as piezoelectric, magnetostrictive, electrostrictive materials, electrorheological, magnetorheological fluids, shape memory alloys, biomaterials...) which are able to respond to any external stimulus such that a change of temperature, pressure, electric or magnetic field by a change of their intrinsic properties (shape, conductivity, polarization, etc). We include also under this word composites or multimaterials whose enhanced characteristics are obtained by combining classical ones by gluing, welding, embedding microstructures or bonding patches. The proper description of the behavior of these materials necessitates models that couple various fields (such that elasticity, plasticity, Navier-Stokes or Maxwell equations) and requires the simultaneous solution of problems known as "multiphysics". Their inherent physical, chemical or thermal properties make them particularly well fitted in the design of adaptive structures.

An *"adaptive structure"* is an integrated system that consists of sensors, actuators and control strategies.

- A sensor senses changes in the environmental conditions (light, pressure, humidity, electric, magnetic field) and induces for example a mechanical, optical, electrical magnetic or thermal response. There are different kinds of sensors according to their use; we can mention strain sensors (to detect mechanical, optical or acoustical properties), displacement sensors (to measure displacement experienced after static loading or vibration excitations) or force and acceleration sensors. The sensors most commonly used are optic fibers and piezo transducers.

- An actuator converts a "driving" energy (in the form of electric, magnetic field, etc) into an "actuating" energy (in the form of heat, radiation or mechanical strain, etc). The most commonly used are the shape-memory alloys (used for their high damping characteristics in passive vibration damping), the piezoelectric, electrostrictive, magnetostrictive, electrorheological actuators.

Applications of adaptive structures are present in almost all domains of industry from telecommunications to aircraft, aerospace, automobile, civil structures, biomechanics and environment (even in extreme environment such that high pressure, high temperature, presence of radiations). Due to recent developments in miniaturization, active transducers are also very popular in various applications of MOEMS (Micro Optical-Electrical-Mechanical Systems), we can cite for example shock absorbers, adaptive or deformable micro-mirrors, micro-accelerometers, capacitive microphones, pressure sensors, micro-positioning actuators, ultrasonic transducers. The applications are in the fields of active vibration suppression or damping, shape control of flexible structures, noise cancellation or attenuation, structural health monitoring, health and usage monitoring systems (mechanical conditions of structures, vibration analysis, damage detection and analysis, with some real-time applications).

The "Smart-Systems" Projects

When looking at the large number of international journals dedicated to smart systems it is not exaggerated to speak about an explosion in the research activity, we can mention for example: Journal of Micromechanics and Microengineering, Journal of Modeling and Simulation of Microsystems, Journal of Micro-electrical-mechanical systems, Journal of Intelligent Materials, Systems and Structures, Journal of Intelligent and Robotic Systems, Active Materials and Smart Structures, etc. The number of related associations and conferences follows the same trend: Transducers, Active Materials and Smart Structures, Structural Control, Adaptive Structures and

Technology (ICAST), SPIE, etc. However, the major part of the published papers and proceedings are presented by engineers and are mostly devoted to the technological aspects of industrial applications. When the question of modeling is addressed, it generally has more to do with a simplified approach obtained with very restrictive geometrical, mechanical or thermo-dynamical assumptions. In general these simplifications are justified, however when refinement is at stake (for example when the objective is in terms of optimization in order to save money and time before manufacturing of prototypes) it is better to go back to the complete mathematical models to properly formulate the problem.

This was at the origin of the application-oriented Research and Training Network "Smart-Systems" aiming at reducing the gap and reinforcing the ties between engineers and applied mathematicians. It was designated to provide efficient mathematical methods and numerical tools dedicated to a better understanding of the behavior of complex systems and of the associated control strategies.

The state of the art presented here has been established by the participants of this European "Smart systems" network, it reflects their major investigations in the three fundamental areas of modeling, optimization and control, and numerical simulation. Let us briefly describe them:

- *Modeling:* The investigation consists in the study of appropriate models for linear and nonlinear coupling effects, for new composite structures (with their nonlinearities induced by hysteresis, phase transitions, contact or presence of cracks) and the associated time-dependent equations. The principal methods used are the following:
  - Asymptotic analysis to establish lower dimensional constitutive laws;
  - Homogenization techniques and multi-scale analysis to establish models for materials with small periodic or nonperiodic micro-inclusions, composite materials, laminate or multilayers sandwich theory;
  - Hemivariational approach for laminates, composites, multiple-phase materials;
  - Quasidifferentiability, variational, quasi-variational inequalities approach, differential inclusions for fracture, contact, impact, friction;
  - Nonlinear analysis of bifurcation and instability, nonlinear partial differential equations in phases transitions, on damage identification.
- *Optimization and Control.* The investigations in this area concern theoretical questions on exact controllability, stabilization, active, passive, hybrid vibration control, optimal shape design (such as the best location of a pattern of sensors, actuators, techniques for nondestructive damage detection, etc) and practical questions about the robustness and accuracy of the design control systems. The principal methods are the following:
  - Active-passive control as exact controllability with piezoelectric actuator, control of hybrid systems, dynamic effects of actuators and sensors made with new materials bonded to or embedded on elastic plates, beams or shells, active vibration control, nonlinear control, time-delayed control;
  - HUM method in control and stabilization;
  - Shape optimization;
  - Constrained optimal control, hybrid active-passive damping .
- *Numerical Simulation.* Two kinds of research have to be brought to a successful conclusion at the same time: at a conceptual level dedicated to theoretical questions such as sensitivity of finite elements (locking, spill-over) and at a practical level dedicated to solve "real-life" applications (such as the simulation of a complete device in order to assess the models retained by checking whether predictions agree with experiments). The principal methods used are the following:

– Reliability and performances of finite elements;
– Virtual Distortion Method to design and control adaptive structures;
– Mathematical programming for optimization of unilateral problems;
– Genetic algorithms as an alternative to deterministic optimization schemes.

1 Modeling and Analysis

The emphasis is on new models for materials with rate-independent behavior (the recent approach uses energy functionals and applies for all generalized standard materials including materials with damage, fracture, plasticity, ferro-electricity or shape-memory effect) in the quasi-static or time evolution setting, contact, growth, etc.). Other nonlinear effects such as interface interaction, existence of a crack or other relevant damages which are of great importance for the design of smart composites are considered too. Another objective is to help designing new multifunctional materials (by homogenization technique and asymptotic analysis) and understand adaptive and biological systems (simulation of complete heart beats, muscle contraction, bone remodeling with mimicking biological actions).

1.1 Smart Materials

1.1.1 Piezoelectric Materials

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1.1.1.1. Orientation / Basic Phenomena:

The piezoelectric phenomenon is twofold:

- Direct Piezoelectric Effect: mechanical deformation → electric field
- Inverse Piezoelectric Effect: electric field or potential difference → mechanical deformation

Basic results: Constitutive laws can be linear or nonlinear, as well as the deformation tensor.

- The linearized constitutive law are

$$\left\{ \begin{array}{lcl} \boldsymbol{\sigma} & = & \boldsymbol{C}{:}\boldsymbol{\varepsilon} - \boldsymbol{e}{\cdot}\boldsymbol{E} \\ \boldsymbol{D} & = & \boldsymbol{e}^T{:}\boldsymbol{\varepsilon} + \boldsymbol{d}{\cdot}\boldsymbol{E} \end{array} \right.$$

  where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon}$ the strain tensor, $\boldsymbol{E}$ the electrical field, $\boldsymbol{D}$ the electrical displacement tensor, $\boldsymbol{C}$ the elasticity tensor, $\boldsymbol{e}$ the piezoelectricity tensor and $\boldsymbol{d}$ the dielectric tensor. Note that other expressions for these constitutive laws can be obtained by "solving" previous relations; for instance previous relations can be rewritten $\boldsymbol{\sigma} = \boldsymbol{C}{:}\boldsymbol{\varepsilon} - \boldsymbol{h}{\cdot}\boldsymbol{D}$ and $\boldsymbol{E} = -\boldsymbol{h}^T{:}\boldsymbol{\varepsilon} + \boldsymbol{b}{\cdot}\boldsymbol{D}$.

- In some applications the nonlinear and history-dependent behavior of piezoelectric materials are of interest to the applications.

References: Fundamentals of piezoelectricity can be found in [1], [2] and a mathematical formulation in [3].

1.1.1.2. Piezoelectric Thin Structures

Piezoelectric materials are generally used as thin components (patches or films) sticked upon or inserted into a structure made of a "classical" material. Thus, it is important to

i) model beams, plates and shells made of a piezoelectric material; for such thin structures, an integration through the thickness leads to different sets of equations depending on the hypotheses made on the behavior of the deformation through the thickness and on the level of these deformations. Such an integration can be just formal or it can be justified mathematically by using the so-called "asymptotic analysis". More details can be found in [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [3], [18],

ii) model systems composed of beams, plates or shells coupled with piezo-electric patches. Practically, these piezoelectric structures are generally used as patches or thin films bonded on, or inserted into, classical structures in order to detect or to generate some deformations, *i.e.* as sensors or actuators.

Mathematical and finite element approximation analysis can be found in [19], [20], [6], [11], [21] [22], [23], [24], [25], while [4], [6], [7] are more oriented toward the applications.

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1.1.2 Electrostrictive Materials

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Orientation / Basic Phenomena: Basic phenomena are similar to those of piezoelectric materials, *i.e.*

- Direct Electrostrictive Effect: mechanical deformation → electric field
- Inverse Electrostrictive Effect: electric field or potential difference → mechanical deformation

but now, the constitutive laws are very non-linear in the domain where they are considered.

Modeling: Let Ω be the domain occupied by the electrostrictive medium. Its boundary $\Gamma = \partial\Omega$ is composed of

$$\Gamma = \Gamma_U \cup \Gamma_F, \quad \Gamma_U \cap \Gamma_F = \emptyset$$

for the mechanical boundary conditions and

$$\Gamma = \Gamma_V \cup \Gamma_Q, \quad \Gamma_V \cap \Gamma_Q = \emptyset$$

for the electrostatic boundary conditions.

Let $\boldsymbol{T}$ be the stress tensor, $\boldsymbol{S}$ be the strain tensor, $\boldsymbol{u}$ be the displacement vector, $\boldsymbol{n} = n_i\boldsymbol{e}_i$ be the unit external normal to Γ, $\boldsymbol{D}$ be the electrical displacement tensor, $\boldsymbol{E}$ be the electrical field, V be the electric potential, q^d and Q^d be the given density of volume and surface loading, $\mathbb{C}$ be the elasticity tensor, and $\mathbb{Q}$ be the electrostrictive tensor.

Mechanical Equations:

$$\begin{aligned}
&T_{ij,j} + f_i^d = \rho \ddot{u}_i && \text{in} \quad \Omega \\
&u_i = u_i^d && \text{on} \quad \Gamma_U \\
&T_{ij} \cdot n_j = F_i^d && \text{on} \quad \Gamma_F \\
&S_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) &&
\end{aligned}$$

Electrostatic Equations:

$$\begin{aligned} & D_{i,i} - q^d = 0 && \text{in} \quad \Omega \\ & V = V^d && \text{on} \quad \Gamma_V \\ & D_i \cdot n_i = -Q^d && \text{on} \quad \Gamma_Q \\ & E_i = -V_{,i} \end{aligned}$$

Constitutive Laws:

$$\begin{aligned} T_{ij} &= A^c_{ijkl} S_{kl} - B^c_{ijn}(\boldsymbol{D}) D_n \\ E_m &= C^c_{mij}(\boldsymbol{D}) S_{i,j} + D^c_{mn}(\boldsymbol{D}) D_n \end{aligned}$$

where the coefficients are given, for example, by

$$\begin{aligned} A^c_{ijkl} &= \mathbb{C}_{ijkl} \\ B^c_{ijn} &= \mathbb{C}_{ijkl} \mathbb{Q}_{klmn} D_m \\ C^c_{mij} &= -2\mathbb{C}_{ijkl} \mathbb{Q}_{klmn} D_n \\ D^c_{mn} &= (\chi^*_{mn})^{-1} \frac{D^s_n}{D_n} \operatorname{arctanh}\left(\frac{D_n}{D^s_n}\right) + 2\mathbb{C}_{ijkl} \mathbb{Q}_{klmn} \mathbb{Q}_{ijpq} D_p D_q \end{aligned}$$

There are many other possibilities including tension control and in some cases variable coefficients, some of them being very nonlinear. For more information, we refer to [26], [27], and [28].

For more details on such modelings, we refer also to [29], [30], [31], [32], [33], [34], [26], [35].

Existence of Solutions: Mathematical analysis of such modelings are still very open, let us cite [27] and [28] for some existence results in the case of rate-independent models introduced by engineers in [10] and [36].

Numerical Approximations: Such models have been approximated by using finite element methods in [37], [38], [39]. See also [40] for a general methodology to prove convergence of numerical schemes in the rate-independent setting.

Applications to Active Control: These electrostrictive properties are used to realize active control of thin structures. Some examples can be found in [33], [41], [26], [35].

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1.1.3 Magnetostrictive Materials

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Orientation / Basic Phenomena: There are two phenomena:

- The *purely magnetic* one: at the microscopic scale, inside of a magnetostrictive material, the magnetization field is distributed into subdomains, each of them being magnetized in a fixed direction. The subdomains are separated by thin walls. When an external magnetic field is applied to such a material, its magnetization field changes. These properties are extensively used in recording devices, for instance.
- The *interactions between mechanical and magnetic properties*

  (mechanical deformation) $\Longleftrightarrow$ (change in the distribution of magnetization field)

  These properties are used in wireless sensors or actuators.

Modeling: Assume that the magnetostrictive material, which occupies a domain Ω, is clamped along a part $\partial\Omega_0$ of its boundary, is loaded by a volume load density $\vec{f}$ in Ω and a surface load density $\vec{t}_{(n)}$ upon the complementary part of its boundary $\partial\Omega_1 = \partial\Omega \setminus \partial\Omega_0$ and is submitted to an external magnetic field $\vec{H}_{ext}$. Under the action of these mechanical and magnetic loads, the material takes deformations characterized by the *displacement field* $\vec{u}$ and a new internal *magnetization field* $\vec{m}$. Both quantities $\vec{u}$ and $\vec{m}$ are the main unknowns of the magnetostrictive problem; they minimize, at least locally, the free energy functional $\phi(\vec{v}, \vec{p})$ whose expression is given by ($\vec{v}$ and $\vec{p}$ are test functions)

$$\phi(\vec{v}, \vec{p}) = \phi_{e\ell}(\vec{v}) + \phi_{co}(\vec{v}, \vec{p}) + \phi_{an}(\vec{p}) + \phi_{ex}(\vec{p}) + \phi_{ma}(\vec{p}) \tag{1}$$

where

$$\phi_{e\ell}(\vec{v}) = \frac{1}{2}\int_{\Omega} \sigma_{k\ell mn}\varepsilon_{k\ell}(\vec{v})\varepsilon_{mn}(\vec{v})d\Omega - \int_{\Omega} \vec{f}\vec{v}d\Omega - \int_{\partial\Omega_1} \vec{t}_{(n)}\vec{v}dS \tag{2}$$

denotes the *elastic energy*, $\sigma_{k\ell mn}$ is the elasticity tensor, $\varepsilon_{jk}(\vec{v})$ is the strain tensor;

$$\phi_{co}(\vec{v}, \vec{p}) = \int_{\Omega} \left\{ e_{ijk}p_i\varepsilon_{jk}(\vec{v}) + \frac{1}{2}\lambda_{ijk\ell}p_ip_j\varepsilon_{k\ell}(\vec{v}) \right\} d\Omega \tag{3}$$

denotes the *coupling energy* between elastic and magnetic effects; coefficients e_{ijk} and $\lambda_{ijk\ell}$ satisfy

$$e_{ijk} = \begin{cases} 0 \text{ for a cubic crystal} \\ e_1 d_i\delta_{jk} + e_2(d_k\delta_{ij} + d_j\delta_{ik}) + e_3 d_i d_j d_k \text{ for a uniaxial crystal} \end{cases}$$

$$\lambda_{ijk\ell} = \lambda_1\delta_{ijk\ell} + \lambda_2\delta_{ij}\delta_{k\ell} + \lambda_3(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})$$

where e_1, e_2, e_3 are piezomagnetic constants, the unit vector $\vec{d} = (d_1, d_2, d_3)$ gives the easy lines of magnetization, the symbol $\delta_{ijk\ell}$ has components equal to one when all indices are alike and zero otherwise, and λ_1, λ_2, λ_3 are magnetoelastic constants;

$$\phi_{an}(\vec{p}) = \frac{1}{2}\int_{\Omega} \chi_{ij}p_ip_j d\Omega \tag{4}$$

denotes the *anisotropic energy* while χ_{ij} are the anisotropic coefficients;

$$\phi_{ex}(\vec{p}) = \frac{1}{2}\int_{\Omega} a_{ij}p_{k,i}p_{k,j}d\Omega \tag{5}$$

denotes the exchange energy. The associate exchange energy coefficient satisfies for instance $a_{ij} = a\delta_{ij}, \quad a > 0$;

$$\phi_{ma}(\vec{p}) = \frac{1}{2}\int_{I\!R^3} |\vec{H}'|^2 dx - \int_{\Omega} \vec{H}_{ext}\cdot\vec{p}d\Omega \tag{6}$$

where the first integral gives the *demagnetizing energy* and the second one gives the *Zeeman energy*. The demagnetizing field $\vec{H}'(\vec{p})$ is solution of

$$\left.\begin{array}{l} \nabla\times\vec{H}' = \vec{0} \\ \nabla(\vec{H}' + \vec{p}\delta_\Omega) = 0 \end{array}\right\} \text{ in } I\!R^3 \setminus \partial\Omega \quad \text{and} \quad \left.\begin{array}{l} \vec{n}\times[\vec{H}'] = \vec{0} \\ \vec{n}[\vec{H}' + \vec{p}\delta_\Omega] = 0 \end{array}\right\} \text{ on } \partial\Omega,$$

so that it is linearly depending on the unknown $\vec{p}$. It is worth to note that

i) In the same spirit, there are more or less other classes of magnetic materials whose modeling are similar enough, for instance ferromagnetic materials;

ii) In magnetism hysteretic effects often occur together with magnetostrictive effects. Such models can be approached by a combination of the rate-independent models, see [42] and [40];

iii) References [43] and [44] are important in the field but with respect to the static case.

For more details on modeling, we refer to [45], [46], [47], [48], [49], [1], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [9], [60], [61], [62], [63], [64], [65], [66], [42].

Existence of solutions: Mathematical analyses in convenient functional spaces can be found in [67], [68], [52], [69], [70], [71], [72], [73], [74], [75], [42].

Numerical approximations: Such models have been approximated mainly by finite element methods in [76], [77], [78], [79], [80], [81], [58], [82], [72], [83], [84], [85], [63], [64], [86], [87], [40].
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1.1.4 Shape Memory Alloys

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The shape memory effect derives its name from the properties of some materials that after a suitable heat treatment the material remembers its original shape despite of prior permanent, plasticity-like deformation. The micromechanical origin of this effect lies in the crystallographic phase transformations between the austenitic phase (higher temperature and higher symmetry) and several variants of the martensitic phase (lower temperature and lower symmetry). The plasticity-like deformations at low temperature are associated with transformations between the different martensitic variants, whereas heating puts the material into the austenitic phase and the subsequent cooling leads to the original distribution of the martensitic variants.

There are already several surveys on these topics and also textbooks, [88, 89, 90]. These effects are usually modeled by introducing internal parameters, here denoted by $z \in \mathbb{R}^N$, which allows us to describe the elastic effects of the different phases. Thus, the potential due to elastic stored energy can be described through the stored-energy density W depending on the linearized strain tensor $\varepsilon(\vec{u}) \in \mathbb{R}^{d\times d}$, $z \in \mathbb{R}^N$ and ϑ, namely

$$\phi(\vec{u}, z, \vartheta) = \int_\Omega W(\varepsilon(\vec{u}), z, \vartheta)\,\mathrm{d}x - \langle \ell(t), \vec{u}\rangle$$

where $\ell(t)$ encodes the exterior loading via volume and surface forces as above. The typical form in the case of linearized elasticity is

$$W(\varepsilon, z, \vartheta) = \frac{1}{2}\mathbb{C}_{ijkl}(z,\vartheta)(\varepsilon_{ij} - \varepsilon^{\mathrm{tr}}_{ij}(z,\vartheta))(\varepsilon_{kl} - \varepsilon^{\mathrm{tr}}_{kl}(z,\vartheta)) + w_0(z,\vartheta),$$

where $\varepsilon^{\mathrm{tr}} \in \mathbb{R}^{d\times d}$ is called the transformation strain tensor. Of course, more general material laws, in particular, the finite-strain case can also be put into this form [91, 92, 93].

The stress-strain relation is given via $\mathbf{T} = \frac{\partial}{\partial\varepsilon}W(\varepsilon, z, \vartheta)$. Moreover, the thermodynamically conjugated driving force associated with z is defined as $X = \frac{\partial}{\partial z}W(\varepsilon, z, \vartheta)$. Since in most applications the heat conduction is a rather fast process (because of small physical dimensions) the temperature may be considered as a parameter which is a given datum, see however [94, 95, 96, 89] for problems where an extra energy equation is taken into account. Thus, we are left with one balance equation for the elastic forces and one equation for the evolution of the internal variable, which take the form

$$\rho\frac{\partial^2}{\partial t^2}\vec{u} = \operatorname{div}\mathbf{T} + \vec{f}_{\mathrm{ext}}(t,x), \quad 0 = \frac{\partial}{\partial \dot{z}}R(\dot{z}) + X.$$

The latter equation, which is sometimes called Biot's equation, includes the dissipation potential R whose derivative gives the internal friction forces arising from changes in z, see [89]. For an example with R being quadratic we refer to [97].

In most application the hysteresis effect is used on relatively slow time scales, hence the inertia effects can be neglected and it suffices to consider the quasistatic setting (with $\rho = 0$). Moreover, to see hysteresis the dissipation is then taken to be rate-independent, which amounts in R being 1-homogeneous, i.e., $R(\alpha\dot{z}) = \alpha R(\dot{z})$ for all $\alpha \geq 0$ and all $\dot{z}$. In that case $\frac{\partial}{\partial\dot{z}}R(\dot{z})$ is replaced by the set-valued subdifferential $\partial R(\dot{z})$. The typical R then is just a norm on $\mathbb{R}^N$ which means that $\partial R(0)$ is the unit ball in the dual norm and the boundary defines the yield surface which X has to reach to initiate phase transformations.

There are many different choices for the variables z. A common situation is that z denotes the local volume fractions of the N different phases, such that z lies in the Gibbs polytope $Z^N = \{ z = (z_i) \in \mathbf{R}^N \mid z_i \geq 0, \ \sum_1^N z_i = 1 \}$ with the unit vectors $e_j \in \mathbb{R}^n$ denoting the pure phase j, see [98, 99, 100, 101, 102].

Another choice for z is made in [103, 104, 105] where $z = \varepsilon^{\text{tr}} \in \{\varepsilon \in \mathbb{R}^{d\times d} | \text{trace}\, \varepsilon = 0, \ \|\varepsilon\| \leq c_0\}$ and

$$W(\varepsilon, \varepsilon^{\text{tr}}) = \frac{1}{2}\mathbb{C}(\varepsilon - \varepsilon^{\text{tr}}) : (\varepsilon - \varepsilon^{\text{tr}}) + c_1|\varepsilon^{\text{tr}}| + c_2|\varepsilon^{\text{tr}}|^2 \quad \text{and } R(\dot{\varepsilon}^{\text{tr}}) = c_2|\dot{\varepsilon}^{\text{tr}}|.$$

An analysis of this model can be found in [106, 107, 108]. A temperature-dependent model is studied anayltically in [109].

The mathematical theory of the finite-strain elasticity in shape-memory alloys was initiated with [110] which led to a huge literature in the field of quasi- and polyconvex functionals, see [111, 88]. However, this theory is mostly restricted to statics. Evolutionary problems are not yet treated systematically for general situations. For situations in space dimension 1 and for only two different phases there is a rich literature in the area of phase transitions, see e.g., [95, 112]. For models with several space dimensions and many different phases there are not many models which are accessible by rigorous mathematics and reliable numerics, like [104, 108]. A more general method is the *energetic formulation* for rate-independent processes which was developed in [98, 100, 113], and which is especially adapted to allow for any number of phases and very general, even nonsmooth constitutive laws, in particular finite-strain elasticity, see [114, 92, 93, 109].

Modeling of microstructures in shape-memory alloys is often done via (gradient) Young measures, in particular in statics, see [110, 111, 88]. Using the energetic formulation these ideas where also transfered to the rate-independent setting in [115, 116, 117, 118], i.e., the temporal evolution of microstructure like twinning and detwinning can be studied analytically in the space of gradient Young measures.

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1.1.5 Optical Fibers

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Optical fiber sensors (OFS) are made of an optical fiber attached to (or embedded in) a structure to be monitored, and of an opto-electronical interrogation unit [119], [120], [121, 122], [123, 124]. They take advantage of the way guided light is influenced by the geometry and stress state of the guide. When an Optical Fiber Sensor is subjected to a mechanical solicitation, its optical length nL, where n denotes the refraction index and L the length of the sensor, varies according both to the geometrical change of the sensor and to the photoelastic effect. Typical simplified expressions involving the refraction index to stress dependence in such a sensor [125] can be found *e.g.* in [126]. Non-linear 3D finite element analysis can be found in [127]. Assuming sufficient kinematical or statical transmission from the surrounding medium to the optical fiber takes place, the accessible light parameters such as phase, polarization or intensity attenuation will inform on the strain or stress state of the structure under monitoring.

The advantages of OFS over other sensors are numerous. The main one is probably the electromagnetic immunity, not forgetting the possibility to carry out measurements along the entire fiber. Moreover, many types of measurements can be carried out with an OFS. As a matter of fact, the sensor results from an interrogation principle combined with a special mechanical packaging of the fiber selecting a physical principle to be mainly active [128]. The European Union has funded several projects such as COSMOS, MONITOR, DAMASCOS and MILLENIUM on optical fiber sensors. Recently, NSF has also funded several projects on this topic in the framework of its special program on *sensors and sensor networks*. Several conferences such as *SPIE meetings on smart structures*, international as well as European workshops on structural health monitoring, and world as well as European conferences on structural control take place regularly and include a large number of talks dedicated to the application of fiber optics technology to new sensing principles. Many kinds of optical fiber sensor systems, including commercial ones, have emerged in aeronautical and civil engineering. The literature is very abundant : see *e.g.* [129], [130] [131], [132], [133, 134], [135], [136, 137], [138]. A few examples are mentioned below.

A first idea consists of detecting a structural crack when the light stops propagating in the fiber [139] or gets strongly attenuated [140].

On the other hand, a pressure sensor is achieved by just laying out a fiber between two staggered combs, the shift corresponding to half the space period of the modes of propagation, typically about 1,3mm for standard multimode fibers. A pressure on the comb yields microbending [141] in the vicinity of contact points, and thus modifies the attenuation of guide. Bridge bearings have been equipped with such sensors in view of real-time truck load monitoring [142], or prestressing monitoring.

An other possible conditioning consists of favoring a uniform but anisotropic stress state, linearly depending on the applied external load to monitor. In this case, photoelasticity induces birefringence and thus modifies the polarization of the light propagating in the fiber. Thus a polarimeter enables one to quantify the stresses in a direction orthogonal to the fiber [143]. This load sensor has been applied to weigh-in motion of vehicles on roads [143]. See also *e.g.* the European project WAVE.

But OFS can also be used to detect chemical species [144] or to measure distributed temperature.

The significance of OFS in the world of sensing appears in [123, 124]. *Pointwise* OFS essentially based upon Bragg-gratings Michelson interferometry and Fabry-Perot cavities [145, 146] are largely commercially available, as they found applications in material sensing and oil industry for examples. They mainly yield local strains, in the same way as conventional strain sensors.

On the opposite, *continuous* OFS are still confidential technologies even in structural monitoring, although the ability to realize fully distributed measurement is a primary advantage of fiber sensors over traditional sensors. Integration of fiber optical sensors in concrete structures may thus offer the opportunity to get a distributed image of the strains in a structure. Distributed sensing relies upon different optical techniques [147] among which are optical time domain reflectometry [148, 149], Raman and Brillouin scattering [150, 151, 152]. Interferometric very long-gauge optical sensors [124, 153] could also be very useful in civil engineering, especially in dynamic evaluations if they were continuously bonded to the structure following a curved arc length : it was proved analytically [154] that two long gauge embedded OFS equipped *e.g.* with a Michelson interferometer can be combined to achieve a low-frequency modal sensor of a slender Euler-Navier-Bernoulli beam provided one OFS is located at a distance from the neutral axis proportional to the curvature of the target mode. Similar results hold for piezo-electrical cables.

A major difficulty that is slowing down the development of distributed optical fiber sensors is the strain-field transfer from the host material to the embedded optical fiber [147]. Materials and their bonding characteristics [155, 156], [157, 158], [159, 160] as well as shape [161] of the sensors strongly influence the difference between the strains in the optical fiber and in the concrete, and more generally, the sensor response. There is a need for a special packaging of the optical fiber meant for attaching the fiber over very long distances and continuously to the concrete [162]. To do so, several solutions have been proposed and analyzed. Anchoring the sensor at both ends [161, 163, 164] requires to pay special attention to debonding issues [161] and material stiffness matching [155, 156, 157]. In view of improving the kinematic continuity between the sensor and the surrounding medium over long distances, wave-like sensors have been proposed and evaluated both by 3D linear finite element analysis and experimentally [165]. In this field, one may predict the strains in the structures for example by just inverting the opto-mechanical coupling. This partially open and computationally intensive approach may boost the use of optical fibers as distributed sensors integrated into the structures.

stateoftheart]

1.2 Other Nonlinear Effects

1.2.1 Delamination and Unilateral Contact Effects

sota]

Orientation / Basic Phenomena: Modeling of interfaces using complete interface interaction laws with vertical branches. It is a subcase of the general nonlinear modeling, with hard nonlinearities concentrated along the interfaces, which is of obvious importance for composite materials and structures. Interface interactions may be of multiphysics nature: mechanical, thermal, electric etc. Delamination means the appearance (initiation) and propagation of a crack at the interface between two laminae in a composite material. The detailed modeling of this highly nonlinear behavior needs consideration of unilateral actions of several kinds: debonding between the two adjacent sides, unilateral contact between the two distinct sides after delamination and corresponding stick-slip effects due to frictional mechanisms. All these phenomena involve variable-structure mechanical models, are highly nonlinear and cause several difficulties to classical mechanical modeling where they are treated in an approximate (trial and error) technique. On the other hand, methods of nonsmooth mechanics provide the general framework for a unified modeling of these effects and of the construction of effective numerical algorithms for the solution of the corresponding problems. In particular, unilateral contact problems, which can be described by monotone, possibly multivalued (with complete vertical branches) laws, lead to variational inequalities. These problems are well-known, can be modeled by means of convex, nondifferentiable potentials (the so-called superpotentials) and refer back to the classical theory of convex analysis and the work by J.J. Moreau, T. Rockafellar etc Debonding effects are more delicate, since they can be modeled by nonmonotone possibly multivalued laws (with falling branches). Their description requires the use of nonconvex, possibly nondifferentiable potentials. The arising problems are known as hemivariational inequalities (after P.D. Panagiotopoulos). The numerical tools for the solution of hemivariational inequality problems and, thus, for the modeling of delamination effects are based on nonsmooth and nonconvex optimization and on the solution of corresponding critical point theories.

Basic Results: Monotone laws (for instance, unilateral contact, classical friction) lead, in principle, to convex problems, while nonmonotone laws (e.g., delamination) to nonconvex ones. The corresponding variational problems are known as variational inequalities and hemivariational inequalities, respectively. The solution methods are based on nonsmooth analysis and optimization, for example the bundle optimization algorithm can be used. The interface laws and their modeling can be considered as a special case of a three-dimensional theory with suitably chosen constitutive laws. The introduction of nondifferentiable, in the classical sense, potentials or dissipation functions leads to the necessity of using more delicate theoretical tools for the study of the arising phenomena. Instead of variational equalities one has variational inequalities. The extension to nonmonotone relations requires the consideration of nonconvex problems. To be more precise let us consider the classical unilateral contact phenomenon as a multivalued, monotone contact law between contact stresses t_n and corresponding boundary displacements normal to the boundary u_{cn} with an initial opening equal to d:

$$-t_n = \begin{cases} 0, & \text{for} \quad u_{cn} \leq d, \\ [0, +\infty], & \text{for} \quad u_{cn} = d. \end{cases} \tag{7}$$

The latter relation can be produced by subdifferentiating the following potential function, which has the form of an indicator function $I_{U_{ad}}$ for the set of kinematically admissible relative displacements: $U_{ad} = \left\{ u_{cn} \in R^1, u_{cn} - d \leq 0 \right\}$,

$$\phi_n (u_{cn}) = I_{U_{ad}} (u_{cn}) = \begin{cases} 0, & \text{for} \quad u_{cn} \leq d, \\ +\infty, & \text{for} \quad u_{cn} = d. \end{cases} \tag{8}$$

Thus, one has the subdifferential unilateral law:

$$-t_{cn} \in \partial I_{U_{ad}}(u_{cn}) \,. \tag{9}$$

Furthermore, by relating the directional derivative of the nonsmooth potential with the classical virtual work principle one gets a variational inequality

$$-t_n \delta u \leq I_{U_{ad}}(u_{cn} + \delta u) I_{U_{ad}}(u_{cn}), \forall \delta u \in R^1, \tag{10}$$

or, taking into account the definition of the indicator function, equivalently, by

$$-t_n \delta u \leq 0, \forall \delta u \in U_{ad}. \tag{11}$$

The above variational inequalities describe local nonlinear effects which are expressed by means of monotone, possibly multivalued, subdifferential laws. They can be integrated along the whole nonlinear boundary, introduced into the principle of virtual work of mechanics and they lead to variational formulations of structural analysis problems which have the form of variational inequalities (see, among others, [166]).

Let us consider the following one-dimensional law which models delamination effects with complete destruction for both compression and tension (or, equivalently expressed, perfect damage for crushing and cracking). For notational simplicity the same traction limit t_1 and delamination yield displacement u_1 are assumed. The nonmonotone law with complete vertical (i.e., falling) branches reads:

$$-t_{cn} = \begin{cases} 0 & \text{for} \quad u_{cn} < -u_1, \\ [0, t_1] & \text{for} \quad u_{cn} = -u_1, \\ c_1 = \frac{-t_1}{u_1} u_{cn} & \text{for} \quad -u_1 < u_{cn} < u_1, \\ [-t_1, 0] & \text{for} \quad u_{cn} = u_1, \\ 0 & \text{for} \quad u_{cn} > u_1. \end{cases} \tag{12}$$

This law can be derived by differentiating the following nonsmooth and nonconvex potential, which can be expressed as a difference of convex functions (i.e., it is a so-called d.c. function):

$$\begin{aligned} \overline{\phi}_n(u_{cn}) &= \min\left\{ \frac{1}{2} c_1 {u_{cn}}^2, \frac{1}{2} c_1 u_1^2 \right\} \\ &= \frac{1}{2} c_1 u_{cn}^2 - \left\{ \begin{array}{cc} 0, & \text{for } u_{cn} < u_1, \\ s_1 u_{cn} + \frac{1}{2} c_1 (u_{cn} - u_1)^2, & \text{for } u_{cn} > u_1 \end{array} \right\} \\ &= \phi_1(u_{cn}) - \phi_2(u_{cn}). \end{aligned} \tag{13}$$

A nonmonotone law of this kind requires the consideration of a nonconvex, possibly nondifferentiable potential and leads to nonconvex variational inequality problems, the so-called hemivariational inequalities (see [167]). Furthermore (see, [168]; [169]) the difference convex structure of (13) can be exploited so that the law (12) can be written as:

$$\begin{aligned} -t_{cn} &= w_1 - w_2, \\ w_1 &\in \partial \phi_1(u_{cn}), \\ w_2 &\in \partial \phi_2(u_{cn}). \end{aligned} \tag{14}$$

The main advantage of using structured (i.e., the d.c. structure) nonconvex approach becomes obvious if one considers the energy optimization approach to nonsmooth mechanics. In that case, elements of d.c. optimization theory and algorithms can be used for the effective solution of nonconvex problems.

References

- Among the Smart Systems Network: The theoretical foundations of nonsmooth mechanics and their relations to unilateral contact modeling and computational nonsmooth mechanics tools have been presented in the monographs [168], [169]. Applications on delamination modeling in composite structures can be found in [170]. [171], [171]. Related results can be found in the edited volumes [172], [173] and [174]. Mathematical analysis and numerical studies of rate-independent models have been done recently and independently by other members of the network. In [175] a rigorous existence proof for a rate-independent model in delamination has been given. Moreover, using numerical simulations it has been demonstrated that the two-sided energy estimate for the time-incremental problem can be used to estimate the quality of the discretized solution.

- Other main references: The theoretical results mentioned in this section can be found in classical publications like [166], [167], [176], [177], [172], [89].

Relevance of Delamination in Smart Structures: Delamination modeling is of importance for the design of smart composite structures, since they are normally subjected to dynamical loading and are prone to fatigue. Furthermore, smart layers may be used to control delamination or even stop it. In the latter context the theory and algorithms of optimal control for structures governed by variational and hemivariational inequalities is relevant.

stateoftheart]

1.2.2 Damage

sota]

Orientation / Basic Phenomena: Damage is due to micromechanical changes of the material subjected to a given loading.

Basic results: The macroscopic modeling of damage is usually based on a continuum model. To avoid mesh dependency sometimes a second order theory is constructed, where the gradient of the damage variable is included. Therefore a non-local model results. In this framework the evolution of damage is governed by a boundary value problem instead of a local evolution law.

References

- Among the Smart Systems Network: In the recent publication [178] a rate-independent model for damage processes under time-dependent Dirichlet data has been presented. This model allows for finite strain as well as for unilateral contact. Existence of solutions under the assumption that the material cannot damage completely but remains a limiting strength (e.g. the stiff fibers can break but the weaker matrix remains intact) has been provided. Finally a discussion with partial results are included for the limit of the energies and the stresses at total damage. The numerical analysis establishing convergence of finite-element schemes for this model is given in [40].

- Other main references: Theory and information of damage mechanics can be found in [179], an application of a continuum model of damage can be found, among others, in [180].

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1.2.3 Viscoelasticity and Viscoplasticity

sota]

Contact problems abound in industry and everyday life. Contact of braking pads with the tire or the tire with the road are just few samples (see the monographs [181], [182] and references therein). Concerning the formulation of a contact problem, there are two main facts: the constitutive law which models the material and the contact condition (friction law, deformable or rigid obstacle, etc), and the external effects due to the process (adhesion, wear, damage, fatigue, etc).

Constitutive laws and friction conditions. The relationship between the stresses in the body and the resulting strains characterizes a specific material the body is made of, and is given by the constitutive law or relation. It describes the deformations of the body from the local action of forces and tractions. Only the framework of small deformations is considered.

Let Ω be a domain in $\mathbb{R}^d$ $(d = 1, 2, 3)$ representing the reference configuration of a deformable body which may be in contact with an obstacle. Let us denote by Γ its boundary which is partitioned into three measurable disjoint parts Γ_D (Dirichlet boundary), Γ_N (Neumann boundary) and Γ_C (contact boundary).

We denote by $\boldsymbol{\sigma}$ the stress field, $\boldsymbol{u}$ is the displacement field and $\boldsymbol{\varepsilon}(\boldsymbol{u})$ represents the linearized strain tensor. The following constitutive laws have been considered in different contact problems:

- A linear elastic constitutive law is given by

  $$\boldsymbol{\sigma} = \mathcal{B}\varepsilon(\boldsymbol{u}),$$

  where $\mathcal{B}$ is a fourth-order elasticity tensor. Contact problems involving elastic materials have been studied there are several decades ago (see the monograph [176] and references therein).

- A viscoelastic constitutive law written as

  $$\boldsymbol{\sigma} = \mathcal{B}(\boldsymbol{\varepsilon}(\boldsymbol{u})) + \mathcal{A}(\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}})),$$

  where $\mathcal{A}$ is a viscosity operator and $\mathcal{B}$ denotes a nonlinear elasticity operator. If we assume that these operators are linear, then the above relation becomes the classical Kelvin-Voigt relation ([183]),

  $$\sigma_{ij} = b_{ijkl}\varepsilon_{kl}(\boldsymbol{u}) + a_{ijkl}\varepsilon_{kl}(\dot{\boldsymbol{u}}),$$

  where b_{ijkl}, a_{ijkl} are the elastic and viscosity coefficients, respectively.

  We notice that, if the memory of the materials is considered, then the viscoelastic term is replaced by ([183], [184]),

  $$\int_0^t a_{ijkl}(t-s)\varepsilon_{kl}(\boldsymbol{u})\, ds.$$

- A viscoplastic constitutive law given by ([185]),

  $$\dot{\boldsymbol{\sigma}} = \mathcal{G}(\boldsymbol{\varepsilon}(\boldsymbol{u})) + \mathcal{B}(\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}})),$$

  where $\mathcal{B}$ still denotes the elasticity operator and $\mathcal{G}$ is a nonlinear viscoplastic operator.

  Rate-type constitutive laws of this type were used to model the behavior of rubber, metals, pastes or rocks (see [185, 186] and references therein). The well-known Perzyna's law, introduced in [183], is a common example of the viscoplastic function $\mathcal{G}$.

Secondly, we turn to describe the different contact conditions. Let us denote by u_n the normal displacement, σ_n the normal stress, $\boldsymbol{u}_\tau$ the tangential displacement and $\boldsymbol{\sigma}_\tau$ the shear stresses. Then, the following contact conditions can be defined on Γ_C:

- Bilateral contact: There is no gap between the body and the obstacle (and thus, $u_n = 0$).

- Normal compliance contact: Describes a reactive foundation. According to [176], [187], it is written as

  $$-\sigma_n = p(u_n - g),$$

  where g denotes the gap between the body and the obstacle and p is a measure of the interpenetration of the surface asperities.

- Signorini conditions: Contact with a rigid foundation. It is written as $u_n \leq g$, $\sigma_n \leq 0$ and the complementarity equation $\sigma_n(u_n - g) = 0$.
- Normal damped response: It corresponds to assume that the reaction of the surface is a function of the surface velocity (i.e., $-\sigma_n = p(\dot{u}_n)$).
- Friction condition: It will be considered of the form

$$\begin{aligned}
&|\boldsymbol{\sigma}_\tau| \leq h(t),\\
&\text{if } |\boldsymbol{\sigma}_\tau(L,t)| = h(t) \Rightarrow \exists\lambda \geq 0\,;\; \dot{\boldsymbol{u}}_\tau = -\lambda\boldsymbol{\sigma}_\tau,\\
&\text{if } |\boldsymbol{\sigma}_\tau| < h(t) \Rightarrow \dot{\boldsymbol{u}}_\tau = \mathbf{0}.
\end{aligned}$$

  We notice that when $h(t) = \mathit{constant}$, the above conditions leads to the classical Tresca's conditions ([183]), and when $h(t) = \mu\, p_T(u_n - g)$ ($\mu > 0$ being a constant) it corresponds to a particular case of the Coulomb friction law. Moreover, if the friction is assumed negligible, then $\boldsymbol{\sigma}_\tau = \mathbf{0}$.

Adhesion and damage (Frémond's models). Processes of adhesion are very important in many industrial settings where parts, usually nonmetallic, are glued together. Different papers have been done concerning this topic. The main idea is the introduction of a surface variable, the bonding field or the adhesion field β, which describes the pointwise fractional density of active bonds on the contact surface. The bonding field is then assumed to vary between 0 and 1, in such a way that when $\beta = 1$ all the bonds are active, when $\beta = 0$ all the bonds are severed and there is no adhesion and, for $0 < \beta < 1$, the adhesion is partial and only a fraction β of the bonds is active.

Different models have been considered in the literature for modeling the evolution of the adhesion field (see the monograph [188] for details). As an example, in [189] its evolution is governed by an ordinary differential equation as,

$$\dot{\beta} = -\gamma_n \beta((-R(u_n))_+)^2,$$

where R is a truncation operator. It describes an irreversible process when only debonding takes place. In [190], the rebonding is allowed and the above differential equation is modified accordingly.

In many materials, such as concrete, there is an observed decrease in the load bearing capacity over time, caused by the development of internal microcracks. The subject is extremely important in design engineering, since it directly affects the usual life span of the structures or components.

There exists a very large engineering literature on material damage. However, only recently models taking into account the influence of the internal damage of the material on the contact process have been investigated mathematically. The models that we considered are based on that introduced by Frémond and Nedjar [191] (see also the monograph [89] for a more detailed description). The new idea involves the introduction of the damage function ζ, which is the ratio between the elastic modulus of the damaged material and the damage-free one. In an isotropic and homogeneous elastic material, let E_Y be the Young modulus of the original damage-free material and E_{eff} be the current one, then the damage function is defined by $\zeta = \frac{E_{eff}}{E_Y}$. It follows, from this definition, that when $\zeta = 1$ the material is damage-free, when $\zeta = 0$ the material is completely damaged, and when $0 < \zeta < 1$ there is partial additional damage and the system has a reduced load carrying capacity, relative to the original one. According to [191], the mechanical model of the damage corresponds to a nonlinear parabolic differential inclusion as,

$$\dot{\beta} - \kappa \triangle\beta + \partial I_{[0,1]}(\beta) \ni \phi(\boldsymbol{\varepsilon}(\boldsymbol{u}), \beta),$$

where $I_{[0,1]}$ denotes the indicator function of the interval $[0, 1]$, κ is the microcrack diffusion constant and ϕ represents the damage source function.

stateoftheart]

1.3 Complex Structures

1.3.1 Modeling of Thin Structures

sota]

The mathematical modeling of thin structures is both a huge and "ancient" field, with some important investigations dating back to the 19th century in the pioneering work of Kirchhoff (see [192]) in particular. Hence, we only mention here the *recent* advances that we see as most directly relevant to the scientific activities and objectives of the network.

Hierarchic approach. As regards the mathematical justification of structural models, a traditional approach consists in considering a model as part of a hierarchy of increasing complexity (e.g. with increasing polynomial order in the transverse direction for plates and shells), and in comparing the model considered with other models of the hierarchy or with the corresponding reference 3D formulation. Within this approach, the following recent works are of particular interest: [193, 194] for obtaining error estimates in the 3D energy norm between various classical models and 3D elasticity (for cylindrical shells); [195] for using a mixed formulation in order to derive error estimates in a straightforward manner (for plates); [196, 197] for defining Sobolev spaces on manifolds and using this concept to revisit classical shell models; [198, 199] for defining and analysing shell models specifically adapted to numerical computations; [200] and [201] for substantiating the relevance of shell models of higher degree than classical ones.

Formal asymptotic analysis. Assuming an expansion of the reference 3D solution as a series in terms of increasing powers of the thickness parameter is also a classical approach in structural modeling. Some recent significant advances have been obtained in this respect, in particular by using variational formulations (instead of strong forms), see in particular [202, 203, 204], [205, 206] and [207] for plates (linear and nonlinear), [208, 209, 210] for shells (also linear and nonlinear), and [211] for incompressible shells.

Asymptotic analysis with convergence results. Following the pioneering work of [212] for linear plates, some most important convergence results have been obtained during the past decades using an asymptotic analysis approach, see in particular [213] for rods, and for shells [214, 215] and [216, 217], see also [218]. The asymptotic results regarding shells, together with concurrent asymptotic results on shell models ([219, 220, 221]), have allowed complete justifications of classical shell models, see [222, 223]. We also point out that convergence results have also been obtained from Γ-convergence theories, see [224] and [225] for linear plates, and [226] for nonlinear plates.

Direct analysis of structural models. Recent progress has also been achieved in the understanding of complex phenomena arising in the asymptotic behaviors of thin structures, in particular for non-standard asymptotic behaviors, see [227], [228], [229], [230], and also for boundary layers [231], [232], [233].

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1.3.2 Elastic Structures with Smart Material Patches

sota]

Orientation / Basic Phenomena: Smart material patches are used as sensors or controls in smart systems. Since the three-dimensional modeling of the possibly coupled (multiphysics) problems is not always feasible, the use of simplified theories and methods of analysis is very important. This is the structural analysis building block which will facilitate investigation of optimal control strategies, optimal design (including design placement of actuators and sensors), inverse analysis etc.

Basic Results: Within the theory of structural analysis the introduction of smart material patches can be treated in several simplified ways. Within elasticity the theory of layered materials and structures can be used. Therefore one can use various theories of composite beams, plates or shells, depending on the application. The consideration of the coupled field effects is more delicate. Let us take as an example a piezoelastic patch bonded on an elastic beam. Either one uses a complete coupling of the two fields and has to solve a coupled elastic-electric problem. Results in this direction can be found in previous sections of this chapter. Or, in a simplified way, a weak coupling is considered. The electric field is solved explicitly and the direct and inverse piezoelectric effects are used in order to construct an equivalent loading (for the actuator) or measurement (for the sensor) expression of the purely elastic structure. Finally only the elastic structure is considered in the modeling of the smart system.

The simplified, decoupled modeling approach has been consistently derived using Hamilton's principle and used in [234], [235]. Some investigations concerning the usage of auxetic materials (elastic materials with negative Poisson's ratio) in smart structures have been presented in [236]. Reference [4] is also of importance.

stateoftheart]

2 Control and Optimization

In order to address the question of stabilization and control of coupled systems encountered in smart structures we first focus on a simplified, linearized model of the scalar heat-wave equations; this gives several results on the non-uniform stabilization, on the sharp dependence of the observability constants and controls with respect to the various parameters entering in the system. The impact of the geometry on the stability properties of solutions in the presence of magnetic and/or thermal effects has also been considered. Theoretical questions arising in the control theory of smart materials (as piezoelectric ones and those whose behavior is modeled by Ginzburg-Landau nonlinear equations) is investigated. Motivated by many engineering applications we also give a first step solution to stabilization and optimal control of structural acoustic problems for noise reduction and we provide robust control feedback laws by investigating recent control methods (with a worst case distribution of expected uncertainties such as cracks or damage).

2.1 Controllability

2.1.1 Controllability of Thin Shells

sota]

The system of differential equations which describes the vibrations of linear elastic thin shells can be reported in the following general form

$$\mathbf{v}_{tt} + \mathbf{A}^m \mathbf{v} + h\,\mathbf{A}^f \mathbf{v} = 0 \tag{15}$$

where $\mathbf{A}^m$, $\mathbf{A}^f$ are the membrane and flexion operators respectively and h denotes the thinness parameter of the shell. The controllability of elastic shells has been intensively studied in the last years especially after the introducing of a constructive controllability method (HUM method) by J.L. Lions [237]. In the pioneer papers [238], [239] the exact controllability of hemispherical shells has been proved. It follows from the existence of an asymptotic gap for the eigenvalues of the Douglis-Nirenberg elliptic operator $\mathbf{A}_h = \mathbf{A}^m + h\,\mathbf{A}^f$. The main theorem states that we have exact controllability for any fixed time T such that

$$T \geq C/\sqrt{h}, \tag{16}$$

where C is a positive constant depending on the characteristic parameters of the material. The analysis is carried out in order to show the role of the geometry and the thinness in the controllability of shells. The exact controllability of shallow spherical shells has been also proved using multiplier techniques; again we get to (16) under the further condition that the shallowness parameter is small enough, the same result has been shown for shells of general shapes; the shallowness condition can be removed, using Carleman estimates (see the references [240], [241], [242], [243], [244]). The time behaviour (16) suggests to look at the controllability of the membrane problem (that is at the limit case $h = 0$). The phenomenology we study concerns the loss of exact controllability in the transition shell-membrane. The reference treatise [245, 246] should be mentioned. In a recent paper [247] an abstract theorem of noncontrollability for linear systems of mixed order, including elastic membrane-shells, has been proved. The non-controllability follows from the existence of the essential spectrum for the membrane operator $\mathbf{A}^m$. The essential spectrum has been characterized and relaxed and partial exact controllability results for membrane approximation has been found ([248], [249], [250]). The analysis has been also extended to the nonlinear case [251], [252], [253]. One may think, as observed in other phenomena, that the exact controllability of membrane-shells follows from the nonlinear terms. The analysis carried out in [254],[255], seems to exclude this possibility; looking also at the phenomenon of the eversion of thin shells to better understand and justify the loss of the exact controllability.

Observability inequalities, and hence some boundary controllability results, for shallow shells of any shapes are consider in [256], [257]. The boundary controllability of thermoelastic plates has been studied in [258].

Several applications involving the controllability of shells can be considered. Here we mention, for example, the paper [259], [260],[261] where an extension of the classical Kirchhoff Model for thin shells is considered and the uniform stability and optimal control of a structural acoustic model with flexible curved wall have been investigated.

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2.1.2 Controllability of Smart Systems

sota]

The extension of the controllability results to more sophisticated materials is not trivial. The dynamics of smart materials is generally described by strongly coupled nonlinear differential systems. For example the motion of ferromagnets is strongly influenced by the dynamics of magnetization vector which is described by a nonlinear parabolic equation

$$\alpha \mathbf{m}_t = \mathbf{m} \times (-\mathbf{m}_t + \mathbf{h}_{eff}), \tag{17}$$

where α is a positive material constant and $\mathbf{h}_{eff}$ is the effective field which usually accounts for the anisotropy energy, the exchange energy, the magnetic field energy and the energy coupling the magnetic and elastic effects. A complete description of the dynamics of ferromagnets requires a further equation (a waves type equation) for the vibration of the magnetoelastic body. The equation (17) which describes the evolution of spin fields in ferromagnets, is the starting point of any dynamic description of micromagnetic processes.

Since the magnetization is represented by an unit vector in the saturated state, it becomes natural the connection with other systems involving harmonic maps as the evolution of some nematic liquid crystals ($\alpha = 0$) and their penalty approximation governed by the Ginzburg-Landau type equation

$$\mathbf{m}^\varepsilon_t - \alpha\, \mathbf{m}^\varepsilon \times \mathbf{m}^\varepsilon_t = \mathbf{h}^\varepsilon_{eff} - \frac{|\mathbf{m}^\varepsilon|^2 - 1}{\varepsilon}\mathbf{m}^\varepsilon, \tag{18}$$

It is well known that in the two-dimensional harmonic map theory, regular initial data can generate singularities in finite time. The main objective is to control the development of singular solutions. Singularities and vortices play an important role in the static and dynamics of various continuous media with order parameters, the analysis of the Ginzburg-Landau functional, originally introduced to study superconducting phase transition, allows to consider various cases of interest in a unified manner (see for example the qualitative and numerical approach in [262],[263],[264],[265],[266]).

Recently controllability results for the Ginzburg-Landau equations have been proved in [267], [268], [269], [270]. The controllability results depend on the positive penalty parameter ε. The controllability for vanishing ε, that is the controllability of harmonic maps, is an open problem still now.

In the framework of the controllability of smart systems we quote also some papers concerning the controllability of *piezoelectric materials*.

Several results in the literature concern with the control of elastic systems obtained by means of piezoelectric patches or devices (even pointwise ones) put inside the structure. We omit here the references on this topic since most of them can be collocated in the frame of the section devoted to the optimization of shapes.

With reference to the model proposed in the previous modeling section, in [271], [272] the controllability of linearized piezoelectric systems has been proposed both for plates and shallow shells. The controllability is obtained by functions applied on the whole boundary. The possibility to control piezoelectric systems only on a part of the boundary could allow to consider also perforated domains and more realistic situations. The interest could be also addressed to the controllability of the nonlinear behavior of piezoelectric materials.

Among the applications we quote transmission problems for systems of piezoelectricity. In [273], [274], [275], [276], systems having piecewise constant coefficients, modeling multilayered piezoelectric bodies, have been considered. For those ones observability and exact controllability results are established.

stateoftheart]

2.2 Vibration Control and Noise Control – Robust Structural Control

sota]

Orientation / Basic Phenomena: The effectiveness of optimal control, which is a crucial component of many smart systems, can be considerably reduced from cracks, damage and other uncertainties. Recent control methods take into account a worst case distribution of expected uncertainty and provide robust control feedback laws. The whole theoretical development in this area is based on extensions of classical optimization such that to take into account uncertainties in the model (i.e., the function to be optimized or/and the constraints) as well as the expression of the optimal control problem with the use of fundamental optimization tools (minimization of a suitable norm with equalities and inequalities) rather than trying to first solve the optimal control problem (using, say, classical Riccati equations) and then trying to use the results.

Basic Results: The technical details are based on (a) a suitably formulated optimal control problem, possibly including H_2 or H_{inf} norms, and (b) a representation of the expected uncertainty, for instance through the linear fractional transformation (LFT) technique. From the theoretical developments one must point out that, as it could be expected, techniques which are based on convex optimization are most suitable. Nevertheless a number of software tools are currently available for the numerical solution of the arising problems.

References

- Among the Smart Systems Network: Recent developments of structural control have been discussed in the book chapter [277]. The concept of robust structural control has been numerically tested [234], [278], [235];
- Other main references: The fundamental material of robust optimization can be found in [279]. The techniques of using convex optimization and reformulating the robust optimal control by using linear matrix inequalities can be found, among others, in [280], [281], [282]. The robust optimal control theory can be found, for example, in [283];

Relevance of Robustness and Uncertainty for Smart Systems: Usually smart devices are used to control the dynamical behavior of systems like vibrations of mechanical structures. Therefore they are exposed to dynamical loadings. On top of this one may have stress concentrations due to the use of composite materials and structures. Under these conditions damage and crack initiation and propagation becomes critical. The mechanical system changes from the nominal one. The questions of structural analysis under uncertainty and, subsequently, robust optimal control so that the effect of uncertainty can be tolerated arise in this framework. stateoftheart]

2.3 Optimal Shape Design

sota]

Orientation / Basic Phenomena: Optimal shape design may be seen a control problem where the control is the boundary or shape of the domain. Therefore, there is a clear analogy between control theory and optimal shape design for evolutionary problems. In fact, using rigorous developments in the context of shape deformations it can be shown that the problem of boundary control is actually the linearizing of that of controlling by means of the shape of the domain. However, as far as we know there are no rigorous results in this direction. This is mainly due to the fact that there is an intrinsic loss of regularity in these nonlinear control problems that makes it difficult to deduce anything about the shape-control problem from the existing boundary controllability results.

Basic Results: A number of results for the sensitivity analysis with respect to shape variations have been published. They include certain classes of domain modifications, like the introduction of small holes, cracks or defects. Furthermore the problem of optimal shape design can be formulated. Classical results of shape design in elasticity have been extended to nonclassical shape design (in the direction of the so-called topology optimization approach) and related fields (like defect identification).

References

- Among the Smart Systems Network: Compilation of results in [284] (some of the crack identification problems in [285] can be seen as shape design problems).
- Other main references: Classical results on shape sensitivity analysis [286], [287], [288], [289], [290], shape design using genetic optimization [291], application on bonded patch design [292] and application on MEMS [293].

Relevance of Shape Design in Smart Structures: In the context of smart system technology, optimal shape design developments are useful to design multiphysics sensors and actuators, to optimally design complete structures or to optimize the location of actuators and sensors for an optimal control application.
stateoftheart]

3 Numerical Analysis

The involvement in the numerics is twofold: on the one hand we continue to improve the efficiency of computational methods (new finite elements for thin structures, boundary layers and contact problems, better understanding of numerical approximation for exact controllability), on the other hand we use our expertise to solve problems applied to smart materials (new models to capture most features of shape memory alloys, ferromagnetic, magnetic steel material), smart and biological systems (composite materials such as laminate plates, material with SMA fibers, MEMS) and structural control (fast stabilization and new approach of impact absorbers)

3.1 Reliable Finite Element Schemes for Thin Elastic Structures

sota]

Many 2-D models involving thin elastic structures can be studied by considering the following *minimization* problem:

$$\begin{cases} \text{Find } u \in V \text{ such that} \\ u = \text{argmin}_{v\in V} \left\{ \dfrac{t^3}{2} a_0(v,v) + \dfrac{t}{2} a_1(v,v) - f(v) \right\} \ . \end{cases} \tag{19}$$

Here t is the thickness parameter ($t \ll 1$), f represents the loads, and V is the space of admissible displacements (incorporating also suitable boundary conditions). Moreover, $a_0(\cdot,\cdot)$ and $a_1(\cdot,\cdot)$ represent internal elastic energy contributions, whose exact expressions differ according with the structure geometry and the adopted model (see [294, 295] and the recent book [296], for instance).

Despite the apparent simplicity of Problem (19), its discretization by finite elements is not at all trivial. A major difficulty is that different numerical strategies are needed for different asymptotic behaviors of the functional in (19), as $t \to 0$; unfortunately, the asymptotic of (19) depends in a very complex fashion on the load f, the geometry of the structure, and the boundary conditions (see [296], for example). Moreover, even when the asymptotic behavior is well-established (e.g. for plate problems), standard finite element procedure (cf. [297]) typically fails the approximation because of the so-called *locking phenomena* (cf. [298]).

Here below we review the most commonly used strategies to design robust finite element schemes.

Reduced schemes. The basic idea is to modify, at the discrete level, the energy term $a_1(\cdot,\cdot)$ in order to reduce its influence. Accordingly, the discrete problem may be written as

$$\begin{cases} \text{Find } u^h \in V^h \text{ such that} \\ u^h = \text{argmin}_{v^h\in V^h} \left\{ \dfrac{t^3}{2} a_0(v^h,v^h) + \dfrac{t}{2} a_1^h(v^h,v^h) - f(v^h) \right\} \ . \end{cases} \tag{20}$$

Above, V^h is a suitable finite element space, and $a_1^h(\cdot,\cdot)$ represents the chosen modification of $a_1(\cdot,\cdot)$. Many efficient methods fall into this class. In particular we mention:

- **The MITC elements**. For plate problems, see [299], [300], [301], [302], [303], [304], and [305], while for the extension to shell problems see [306], [307] and [308].
- **The Linked Interpolation Technique**. It has been studied in [309], [310], [311], [312], [313] and [314] in the framework of elastic plate problems. Its generalization to the case of *piezoelectric* plates has been considered in [14].

Stabilized formulations. The main idea, first proposed in [315] for low-order elements, consists in solving the discrete problem

$$\begin{cases} \text{Find } u^h \in V^h \text{ such that} \\ u^h = \text{argmin}_{v^h\in V^h} \left\{ \dfrac{t^3}{2} a_0(v^h,v^h) + \dfrac{t(h)}{2} a_1(v^h,v^h) - f(v^h) \right\} \ . \end{cases} \tag{21}$$

Here, $t(h) \approx \dfrac{t^3}{t^2+h^2}$ is a sort of 'thickness modification', which may have a stabilizing effect. In the context of the augmented formulations, several interesting variants and generalizations have been proposed, among others, in [316], [317], and especially [318] for shells in a bending-dominated state.

PSRI Technique. The 'Partial Selective Reduced Integration Technique' (PSRI) essentially consists in re-writing Problem (19) as

$$\begin{cases} \text{Find } u \in V \text{ such that} \\ u = \text{argmin}_{v \in V} \left\{ \frac{t^3}{2}\big(a_0(v,v) + a_1(v,v)\big) + \frac{t-t^3}{2} a_1(v,v) - f(v) \right\} \ , \end{cases} \tag{22}$$

and in considering the discrete problem

$$\begin{cases} \text{Find } u^h \in V^h \text{ such that} \\ u^h = \text{argmin}_{v^h \in V^h} \left\{ \frac{t^3}{2}\big(a_0(v^h,v^h) + a_1(v^h,v^h)\big) + \frac{t-t^3}{2} a_1^h(v^h,v^h) - f(v^h) \right\} \ , \end{cases} \tag{23}$$

Therefore, the energy term associated with $a_1(\cdot,\cdot)$ is split into two parts: the first one is exactly integrated, while the second one is reduced. This strategy, proposed in [319] and [320], allows for a wide choice of energy reduction procedures and prevent from the occurrence of spurious modes. Several plate elements taking advantage of this approach have been studied in [321] and [322]. Elements for particular shells in a bending-dominated state have been proposed in [323].

Other Schemes. Many other methods, some of them based on a combination of the techniques described above, have been presented in the literature. Here we mention [324], [325], [326], [327], [328], [329], [330], and [331]. Moreover, we cite the recent elements in [332], [333] and [334], which take advantage of the Discontinuous Galerkin machinery.

A-posteriori Error Estimators. Few *a-posteriori* error estimators have been proposed and analyzed for plate problems: we cite the paper [335] for the Arnold-Falk element (cf. [324]); the works [336] and [337] for elements using the PSRI Technique; the paper [338] for the MITC-type element detailed in [302]; the manuscript [339] for the Linking Technique. Another related work, more engineering-oriented, is [340].
stateoftheart]

3.2 Reliable Finite Element Schemes for Viscoelastic and Viscoplastic Structures

sota]

Contact problems with viscoelastic materials Viscoelastic contact problems have been studied during the last decade from both mathematical and numerical point of views. Since the problem can be written in terms of the velocity field, the variational formulation leads to a nonlinear variational equation (when the contact is with a deformable obstacle and without friction) or a nonlinear variational inequality of the second kind (when friction or rigid obstacle are considered). As an example, the following variational formulation corresponds to an abstract problem including some of the frictionless problems considered.

Problem P. *Find* $u : [0,T] \to V$ *such that*

$$\begin{aligned} & u(t) \in U, \quad (\dot{u}(t), v - u(t))_V + (Bu(t), v - u(t))_V \\ & \qquad\qquad\qquad \geq (f(t), v - u(t))_V \qquad \forall\, v \in U, \quad a.e.\, t \in (0,T), \\ & u(0) = u_0, \end{aligned}$$

where V is a Hilbert space and $U \subset V$ is a non empty closed convex subset of V. B is a nonlinear operator (which represents the elastic part in the examples). Dynamical problems are also studied. The only difference is that inertia term can not be neglected in the equation of motion. The mathematical analysis is provided for all the problems using classical results of evolutionary variational inequalities and fixed point arguments.

Since contact problems with viscoelastic materials were presented in [341], the results shown there where extended to the case including the adhesion ([342, 343, 344, 190]) or the damage ([345, 346, 347]). In all these papers, our main contribution concerns the numerical analysis of a fully discrete problem. This is obtained using the finite element method to approximate the spatial variable (as an example, usually continuous piecewise affine functions were employed) and an Euler scheme to discretize the time derivatives. Error estimates were proved and, under suitable regularity assumptions, the linear convergence of the algorithm is deduced. We notice that this fully discrete scheme was solved using a penalty-duality algorithm introduced in [348] and studied precisely in [349, 350] for the elastic case. As an example, the following fully discrete scheme corresponds to the discretization of the above Problem P.

Problem P^{hk}. *Find $u^{hk} = \{u_n^{hk}\}_{n=0}^N \subset V^h$ such that:*

$$u_0^{hk} = u_0^h,$$

and for $n = 1, \dots, N$,

$$u_n^{hk} \in U^h, \quad ((u_n^{hk} - u_{n-1}^{hk})/k, v^h - u_n^{hk})_V + (Bu_n^{hk}, v^h - u_n^{hk})_V$$
$$\geq (f_n, v^h - u_n^{hk})_V \qquad \forall\, v^h \in U^h,$$

where u_0^h is an appropriate approximation of the initial condition u_0. Here h denotes the spatial discretization parameter and k is the time step. Since the implicit Euler scheme is applied, a Newton method was used to solve the above discrete problem.

Moreover, in [351] the thermal effects were also included, and the one-dimensional case, from modeling and numerical point of views, was studied in [352] (rods) and [353] (beams).

We also notice that contact problems with long-term memory materials were studied in [354, 355, 356, 357] included in the recent PhD Thesis [358].

Contact problems with viscoplastic materials. As in the previous section concerning viscoelastic materials, variational and numerical analysis were performed for different quasistatic viscoplastic contact problems. In this case, since the stress field can not be written in terms of the velocity or displacement fields, a fixed point argument is used for obtaining the existence of a unique weak solution and for deriving error estimates (see [359, 360, 341]). As in the previous section, the finite element method was employed to approximate the spatial variable and finite differences (Euler's method) to discretize the time derivatives.

Recently, quasistatic viscoplastic contact problems with damage or hardening have been studied ([361, 362, 363]). Moreover, in [364] the adhesion was also included.

stateoftheart]

4 Simulation and Industrial Applications

4.1 Magnetostrictive and Ferromagnetic Materials

Magnetostrictive behaviors of materials can be used in a very large spectrum of applications. Without exhaustiveness, we can mention three major fields of applications

4.1.1 Applications Based on the Interaction between Magnetic and Mechanical Properties

sota] These interactive properties are mainly used to construct really efficient wireless sensors and actuators in order to actively control structures. Some associated applications can be found in [80], [365], [366], [367]. stateoftheart]

4.1.2 Applications to Thin-Films and Magnetic Recording

sota] Magnetic recording represents by far the most rapidly growing application of magnetic materials. An extensive discussion about such applications can be found in [368], [59, section 6.4]. Likewise section 6.5 of this reference discusses thin-film devices. Another contribution in this field was made by [369]. stateoftheart]

4.1.3 Applications to Non-Destructive Control

sota] In order to improve production methods and material quality, material scientists try to use the connection between various microstructural aspects and magnetization curves. Among design tools and nondestructuve control, there is a need for reliable hysteresis models and a good understanding of phenomena like Barkhausen noise. For extended discussions on these fields, we refer, to the books by [60] and [59]. More specialized contributions were made, for instance, by [370], [371], [372], [373], [374]. stateoftheart]

4.2 Shape Memory Alloys

sota]

We first wish to note how the richness of theoretical and modeling contributions does not correspond to a richness of studies relative to time-discrete algorithmic solutions. Due to the importance of proposing models viable of a robust numerical solution, we briefly review the major contributions in this area. However, in most of the cited reference, unless stated, the authors do not give too much details regarding the numerical schemes, presenting in general also a limited or non-exhaustive set of numerical investigations.

- Brinson and Lammering [375] implement in a large deformation finite element context the 1D model presented in Reference [376].
- References [377, 378, 379] discuss a finite-element implementation of the model presented by Raniecki et al. [380, 381].
- Trochu et al. [382, 383] present a discussion on the numerical implementation of a three-dimensional model based on the dual-kringing interpolation method.
- Govindjee and Kasper [384, 385] pay attention to computational aspects relative to a specific 1D constitutive model, addressing in details the loading-unloading conditions and possibly pathological situations.
- Qidwai and Lagoudas [386] use return mapping algorithms for the numerical implementation of the model discussed by Boyd and Lagoudas [387]. However, the implementation neglects the reorientation process for martensitic variants.
- Souza et al. [103] propose an interesting model together with a detailed discussion of an integration algorithm, cast within the return map family. The authors test the model on some simple one-dimensional problems (both in the superelastic and in the shape-memory effect range) and on a more complex three-dimensional problem, involving a non-proportional loading path. In particular, a comparison with experimental results shows the good performance of the proposed model as well as the ability to reproduce the martensite reorientation. An extension of the cited model to the case of non symmetric response in tension and compression together with an accurate discussion of a robust integration algorithm has been recently proposed by Auricchio and Petrini [104, 105, 388].
- Auricchio et al. [389, 390, 114, 107] discuss the numerical implementation of a three-dimensional finite-deformation superelastic model developed within the generalized plasticity framework. Auricchio and Sacco [391, 392] detail numerical schemes for two beam models, respectively able to describe the superelastic effect and able to describe both the superelastic and the shape memory effect.

- Convergence of space-time discretization for general rate-independent material models are treated in [40]. This includes models for the hysteretic behavior of shape-memory materials at small and at large (but moderate) strain.
- Some theoretical results concerning a time-discretization scheme for a model proposed by Frémond have been developed in [393] and [394].
- Computational aspects of microstructure formation and evolution in the three-dimensional setting is treated in [101, 395, 117, 118].

stateoftheart]

4.3 Biomechanics

sota]

Biomechanics of skeletal muscles: from 1-D to 3-D continuum models

Skeletal muscles have a complex behavior: they are essentially incompressible, they certainly are not isotropic, and, if a single fiber direction exists at each point, they probably may be considered transversely isotropic; they are highly nonlinear with passive hyperelastic stress-stretch relations; strain rate effects also exist, as given, namely, by the hyperbolic relation between stress and strain rate (Hill's equation); and, most important of all, activation processes may take place along the muscle fibers due to neural (electrical) stimulation: in the absence of neural stimulation, the actine and myosine sliding filaments of the muscle fibers freely slide relative to each other, while the neural stimulation gives rise to complex electro-chemical processes that leads to an increase in Calcium ion concentration and allows for the formation of cross bridges between the actine and myosine filaments, increasing thus the muscle activation level and its capacity for stress generation.

As observed by Humphrey [396] "*The long-standing dogma in muscle mechanics comes from the classic works by A.F. Huxley [397] and A.V. Hill [398]. That is, it has long been thought that the mechanics of muscle contraction is one dimensional, described by a tension T.*"

Hill ([398],[399]) proposed a 1-D mechanical model composed by three elements: an (active) contractile element in series with a (passive) elastic element, both of them in parallel with another (passive) elastic element. It is the contractile element that is responsible for the free change in length of the muscle (when non-activated) and the force generation in the muscle (when activated).

Huxley ([397],[399]) proposed a 1-D cross-bridge model of muscle contraction which states that, during contraction, a fraction of all cross-bridges is attached, and every attached cross-bridge has its own dimensionless attachment length ξ. The distribution of attached cross-bridges with respect to their length is given by the function $n(\xi, t)$ and the rate of change of this distribution can be expressed by a partial differential equation (Huxley equation). The active muscle stress can be determined from the distribution of attached cross-bridges $n(\xi, t)$.

1-D muscle actuators have thus been incorporated in most simulations of the musculo-skeletal system that have been performed during the last decades. Concepts and computational methods of multi-body dynamics have been used, with applications that range from sports to orthopedics and rehabilitation. Simplified instantaneous actuators having only maximum and minimum force inequality constraints were initially used with "inverse dynamic calculations" and with "static optimization" to estimate instantaneous values of the muscle forces in (parts of) the redundant musculo-skeletal system, for given skeletal motions. In other simulations, the proposed 1-D muscle models evolved from the original work of Hill (Zajac *et al.* [400],[401], Pandy *et al.* [402], [403], Winters [404]). In some cases, the determination of the unknown kinematic, force, activation and neural stimulation variables in a time interval involves "forward dynamic calculations" and requires the resolution of an "optimal control" problem.

The 1-D muscle model proposed by Zajac [400] has the longitudinal muscle stress equal to the sum of the stresses in the contractile element and in the parallel element $T = T^{CE} + T^{PE}$. The stress T^{CE} in the contractile element is of course equal to the stress in the series elastic element

T^{SE} and is given by the product of: (i) a function of the muscle stretch which has the typical bell or inverted parabola shape, with maximum value at the muscle rest length, (ii) a function of the strain rate of the contractile element, and (iii) an activation variable. On the other hand the stresses in the series and the parallel elastic elements are highly nonlinear functions of their elongations and essentially vanish in compression. Putting all this together, the governing equation for the contraction process can be written as a first order differential equation that expresses the time rate of change of the muscle stress, with the values of the muscle stress, the muscle deformation rate and the activation:

$$\dot{T} = \mathfrak{T}(T, \lambda, \dot{\lambda}, a)$$

In addition, the time lag between the neural signal, represented by a scalar control variable $u \in [0, 1]$, and the corresponding muscle activation $a \in [0, 1]$ is governed by the dynamics of the Ca^{2+}, and is represented at macroscopic level by another first order ordinary differential equation: $\dot{a} = \mathfrak{A}(a, u)$.

The work of Huxley has also been continued by various authors, namely by Zahalak ([405],[406]), who proposed the Distribution-Moment model (DM model). In this model the specification of a particular shape for the distribution $n(\xi, t)$ of attached cross-bridges simplifies the mathematical structure of Huxley's theory leading to a system of ordinary differential equations governing the time rate of change of a finite number of distribution moments ($Q_i(t) = \int_{-\infty}^{+\infty} \xi^i n(\xi, t) d\xi$ is the i^{th} distribution moment). The 0^{th}, 1^{st} and 2^{nd} moments are, respectively, the fiber stiffness K, the fiber stress T and the elastic energy U stored in the muscle fibers, so that the corresponding ordinary differential equations are

$$\dot{T} = \mathfrak{T}(K, T, U), \quad \dot{K} = \mathfrak{K}(K, T, U), \quad \dot{U} = \mathfrak{U}(K, T, U).$$

An additional first order differential equation exists also for the activation process.

The simulation of the deformation of skeletal muscles as 2-D or 3-D solids started with motivations that ranged from pure animation and entertainment purposes (without much preoccupation on the details of the underlying physical phenomena) to medical applications with muscle structures that are not reducible to 1-D (for instance, the muscles of the pelvic floor, the diaphragm muscle that separates thorax and abdomen).

The main difficulty in the passage from 1-D muscle models and computations to the 2-D and 3-D cases has been the lack of reliable experimental data on the transversal behavior of the skeletal muscles. The approach that has been followed by most authors consists of adding to the much studied longitudinal behavior of the muscle fibers additional contributions related to an embedding matrix and to incompressibility or quasi-incompressibility:

$$\mathbf{t} = \mathbf{t}_{matrix} + \mathbf{t}_{incomp} + \mathbf{t}_{fiber}.$$

The fiber contribution for the stress tensor $\mathbf{t}$ has the form $\mathbf{t}_{fiber} = T_{fiber}\mathbf{n} \otimes \mathbf{n}$, where T_{fiber} is the scalar tension on the fiber and $\mathbf{n}$ is the unit vector of its current direction. The contribution $\mathbf{t}_{incomp}$ has the form $-p\mathbf{I}$, where p is an hydrostatic pressure and $\mathbf{I}$ is the identity, when it is due to perfect incompressibility; it has a form of the type $\frac{1}{D}\frac{d\phi}{dJ}$, where D is a compressibility constant and ϕ grows with $J-1$ in order to penalize the volume change, and grows unboundedly as $J \to 0$ so as to prevent material collapse. The matrix contribution $\mathbf{t}_{matrix}$ is typically an isotropic hyperelastic contribution of a form similar to other forms adopted for soft tissues.

In the 3-D Hill-type constitutive relation for the skeletal muscles adopted in Martins *et al.* [407] the isotropic matrix contribution has the same (exponential) form as the one adopted by Humphrey and Yin [408] for the passive behavior of cardiac muscle, but with material parameters obtained in some compression experiments with skeletal muscles available in the literature. The fiber stress T_{fiber} is the sum of the nonlinear elastic contributions of the parallel and the series elastic elements [401]. For quasi-static computations, the rate effects are not taken into account and, concerning activation, it enters the computational model by means of a prescribed (input) time history of a strain-like quantity associated with the contractile element. A different version of this model has been recently adapted by d'Aulignac *et al.* [409] for shell analysis. Again, rate effects are not taken into account, but, this time, the input is indeed the time history of the activation variable, $a(t) \in [0, 1]$, the series elastic element is not considered, and all passive nonlinear elastic behavior is assigned to the isotropic matrix.

A few other models have been proposed for skeletal muscles.

The 3-D model proposed by Kojic *et al.* [410] is a 3 element Hill-type model, with the major simplification that all passive elastic behavior is linear elastic and isotropic and is assigned only to the matrix (i.e., the linear elastic matrix plays the role of a parallel elastic element). The incompressibility constraint essentially is not taken into account (Poisson's ratios $\nu = 0.45$ or 0.3 are used), and the time history of the activation variable is also the input for the analysis. On the contrary, incompressibility is taken into account in the 3-D Hill-type model proposed by Johansson [411] by a mixed displacement-pressure formulation. The model input is again the time history of an activation variable. But this model does not consider a series elastic element and assigns all passive nonlinear elastic behavior to the matrix that embeds the fibers. These behave thus simply as the contractile element.

The Distribution-Moment model of Zahalak has also been used in the contractile muscle fibers of the 3-D constitutive models proposed, namely, by Gielen [412] and by Oomens *et al.* [413] . In the latter case, the passive muscle behavior is modeled with the most simple incompressible hyperelastic model, the Neo-Hookean material, which, in fact, is typical for rubber-like materials.

In what concerns the difficulty mentioned earlier of finding appropriate experimental data for the construction of 3-D muscle models, it is worth to mention the paper by Bosboom *et al.* [414], though it does not provide much data. Earlier suggestions that the muscle tissues may exhibit multi-axial effects upon contraction and relaxation can be found in Strumpf et al. [415]. It is also important to notice that Zahalak *et al.* ([406], [416]) proposed a three-dimensional generalization of the classic Huxley cross-bridge theory, i.e. a new Huxley-type rate equation for the bond-distribution function, which accounts for the effects of non-axial deformations on the active axial stress and also on non-axial (e.g. shearing) components.

The modeling of cardiac muscles can also be performed in a framework derived from the above Huxley and Zahalak approaches, see in particular [417]. However, most existing models of cardiac myofiber contraction mainly rely on heuristic approaches and macroscopic experimental testing, see e.g. [418] and references therein.

stateoftheart]

4.4 Vibration and Control

sota]

Orientation / Basic Phenomena: Reduction of structural vibrations can be achieved by means of passive, semi-active or active control mechanisms. The whole system can be defined as smart (or intelligent) system. The realization of the sensors and actuators can be based on smart materials, layers or composites.

Basic Results: The most applications are based on classical layered or composite material theory or even on the homogenization concept with suitable combination with control strategies (if applicable, since passive control does not require the optimal control methodology). The proposal of innovative designs and applications is a challenge for engineers and material scientists.

References among the Smart Systems Network: Active control applications of beams equipped with piezoelectric sensors and actuators have been presented in [234], [235]. Other recent applications of vibration control on frame structures have been discussed in [277]. Stochastic loadings have been taken into account in [419].

stateoftheart]

4.5 Impact and Control

sota]

Attenuation of impact is a very important engineering problem. Many structures are exposed to a risk of heavy dynamic loads, which vary significantly in their nature. The problem covers a wide spectrum of cases: from single loads of short duration, cyclic impact loads (e.g. cracking ice), explosions to impacts with hyper velocities. A broad review of the subject can be found in [420], [421], [422].

Modeling of impact generally involves solving highly nonlinear initial boundary-value problem. The nonlinearities are related to:

- large strains, large displacements and contact - geometrical nonlinearities
- plasticity, rate-dependent material models, fracture etc. - material (physical) nonlinearities
- nonlinear behavior of fluid and fluid-structure interactions

Usually the solution is obtained by applying finite element method with explicit time integration. Solving large scale, nonlinear FE models is a very computationally demanding task, therefore analytical or semi-empirical methods are often utilized. One of them is based of assuming rigid-perfectly plastic model of material and considering different models of damage mechanism (kinematics of rigid parts of the material connected by plastic hinges). Theoretical results for thin-walled structures, presented e.g. in [423] agree very well with experimental results. Typically, the structural designs focus on passive energy absorbing systems. These systems are frequently based on the aluminum and/or steel honeycomb packages (see [424]).

Although they are characterized by a high ratio of specific energy absorption, passive energy dissipaters are designed to work effectively in pre-defined impact scenarios. Performance of passive systems can be improved by means of structural size and shape optimization. Those methods because of the scale of models combined with high nonlinearity of the problem, often implement surrogate models (e.g. method of surface of representation). Various formulations of crashworthiness-based structural design problem are presented in papers [425], [426], [427], [428], [429], [430] , [431]. Although topological optimization provide very good results in terms of crashworthiness, the solutions are very sensitive to changes in loading conditions.

Requirements for optimal energy absorbing systems may be stated as follows:

- the system must dissipate the kinetic energy of an impact in a stable and controlled way,
- displacements must not exceed maximal allowable values
- accelerations and forces of the impact should be minimized

Almost any properly designed passive systems fulfill the first two of the requirements. The third one, because of their constant constitutive force-displacement relation, can be realized only to some extent. Therefore, in most cases, commonly applied passive protective systems are only effective for a single impact scenario.

In contrast to the standard solutions, the approach proposed in [432], [427], [433] focuses on active adaptation of energy absorbing structures (equipped with sensor system detecting impact in advance and controllable semi-active, smart-material based dissipaters, so called structural fuses) with high ability of adaptation to extreme overloading. In order to minimize the consequences of the dynamic load, a process of structural adaptation to an impact should be carried out. The process consists of the two following, subsequent stages:

- Impact detection

  Impact detection is provided by a set of sensors, which respond in advance to a danger of collision (e.g. radar, ultrasonic devices) or are embedded into the structure within a small passive crush zone (e.g. piezo-sensors). Estimation of the impact energy is then based on an initial deformation of the passive zone.

- Structural adaptation

  The signal from the system of sensors must be directed to a controller unit, which selects an optimal distribution of yield forces in active zones containing elements, equipped with structural fuses consisting of a stack of thin, shape memory (SMA) alloy washers. The yield force in the active element depends on a friction force generated in the fuse by activating a different number of washers.

Similar approach is implemented in the design of adaptive shock absorbers in aircraft landing gears (http://smart.ippt.gov.pl/adland). After estimation of kinetic energy of the aircraft, the data is transmitted to a controller unit, which selects optimal, precomputed control strategy. Hydraulic damping force can be actively alternated during the impact by changing fluid flow regime inside the damper. The solution is based on magnetorheological fluids (fluids sensitive to a magnetic field) or piezoelectric elements, which change the size of the orifice in the damper.
stateoftheart]

4.6 Structural Health Monitoring

sota] *Introduction*

A concise and informative definition of Structural Health Monitoring can be found in [434]: "The process of implementing a damage detection strategy for aerospace, civil, and mechanical engineering infrastructure is referred to as Structural Health Monitoring (SHM). The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of the system's health." The damage state of a system can be described as a five-step process as discussed in [435]. The damage state is described by answering the following questions:

1. Is there damage in the system (existence)?
2. Where is the damage in the system (location)?
3. What kind of damage is present (type)?
4. How severe is the damage (extent)?
5. How much useful life remains (prognosis)?

Many SHM methods originate from the Non-Destructive Testing (NDT) methods i.e. ultrasonic testing, acoustic emission, radiographic testing, magnetic particle inspection, eddy currents, penetrating testing, and holography, which are successfully applied in the industry for local detection of flaws in structural components. The objective of SHM is to create a monitoring system (possibly for the whole structure) able to track changes in structural health continually and raise appropriate alerts if a defect is detected. Methods for identifying structural damage can be roughly split into high- and low-frequency-based excitation methods. As examples, high-frequency-based methods are used in aerospace for crack identification in a wing, whilst low-frequency-based methods are used in civil engineering for examining stiffness degradation of a bridge.

Smart sensors in SHM

There is a variety of sensors used in SHM. Perhaps the most commonly used are piezo-electrics because of their low price and both actuating and sensing capabilities. An in-depth presentation of piezo-electric sensors is given in [436]. The physical quantities, which can be directly measured by piezo-sensors are force, strain, pressure and acceleration. Another widespread sensor with a growing number of applications in SHM is optical fiber. An extensive overview of various types of optical fiber sensors is presented in [437]. The major division line is among the interferometric sensors for outside application and fiber Bragg grating sensors for inside application (embedded in the structure). Other sensors used in SHM are: electromagnetic transducers, Micro-Electro Mechanical Systems (MEMS) and laser vibrometers.

System identification

An important stage of SHM analysis, carried out before the commencement of identification procedure, is system identification aiming to determine a reliable reference state. Auto-regressive moving average family models [438], [439] and stochastic subspace identification methods [440], [441] are used for identification of system parameters from experimental data. An issue closely related with system identification is model calibration for the methods where a numerical model is used.

High-frequency methods

High-frequency-based (above 20 kHz) methods are focused on precise identification of a relatively small defect in a narrow inspection zone. A comprehensive review of high-frequency-based methods is given in [442]. All of the methods are based on the phenomenon of elastic wave propagation.Acoustic emission (AE) [443], [444] utilizes structure-borne stress waves generated by internal material defects under external load applied. It is a passive method - no excitation is required. The remaining methods require high-frequency excitation. Ultrasonic testing (UT) [445] relies on the transmission and reflection of bulk waves and utilizes various phenomena (e.g., wave attenuation, scattering, reflections, mode conversions, energy partitioning) for damage detection. The most frequently used damage detection method based on guided (between two boundaries of a plate) ultrasonic waves is Lamb wave inspection [446], in which structural defects are usually identified by examining wave attenuation and mode conversions. Acousto-ultrasonics [447], [448] is another method combining elements of AE, UT and Lamb wave inspection. It uses an impulse excitation resulting in a number of mixed propagating wave modes. Finite Element Method has been extensively used for modeling wave propagation. However there is a problem of accuracy and efficiency in the high frequency regime. To overcome those, Spectral Element Method [449] has been developed providing high accuracy for a relatively coarse mesh thanks to defining the element stiffness matrix in the frequency domain.

Low-frequency methods

In contrast to high-frequency methods, low-frequency-based (below 5 kHz) ones use vibration-sensitive parameters to identify significant damage in a relatively broad inspection zone. Most low-frequency vibration-based methods for damage identification require a finite element model. Several damage-sensitive parameters of the model are selected for diagnosing structural health. The damage identification procedure consists of updating the model parameters to best match an experimental response of a damaged structure. A resulting inverse problem has to be solved. Natural frequencies and mode shapes are often used for assessing damage [450], [451]. Anti-resonance frequencies can improve the accuracy of damage predictions [452]. Modal curvature was identified as a damage-sensitive parameter [453]. The SHM algorithms also contained modal strain energy [454] [455] and modal kinetic energy [456] as parameters. Efficiency improvement from model reduction is an important issue in model updating [457]. Fast reanalysis methods proved their effectiveness in SHM [458]. An interesting alternative to inverse methods in damage assessment is Direct Stiffness Calculation [459]. Except for Finite Element Method, Boundary Element Method [460], [461] has been successfully used for low-frequency damage identification as well.

Artificial intelligence in SHM

Most structural health monitoring methods include Artificial Intelligence (AI), involving: wavelet transformations [462], [463], artificial neural networks [461], [464], [465], [466], genetic algorithms [467], filter-driven methods [468], [460], and statistical analysis [469], [470], [471]. The AI methods are employed at the stage of signal processing, when the response (signature) of the intact structure is confronted with a damaged response. The AI tools are efficient, although they disregard the physical interpretation of the analyzed responses. Case-based reasoning (CBR) [472] is a worth-noting method, combining wavelet transformation with Kohonen-like neural networks (self-organizing maps). However, the use of knowledge-based approaches (such as CBR) has not been extensively exploited for damage detection yet.

Applications

SHM methods have found application mainly in aerospace - aircrafts [473], satellites [474] and civil engineering -bridges [457], [458], [475], [476], [476], buildings [462], [477]. Other applications include water networks [478], pipelines [479] and composites [480]. Relevant applications and techniques can be found in publications related to parameter and crack identification problems, e.g., [481], [482], [460] and [483].

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5 List of References

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