Department for Mathematics and Computer Science
Free University of Berlin
Prof. Dr. V. John, john@wias-berlin.de
Hanne Hardering, harderin@zedat.fu-berlin.de

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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 06

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Let $f(x) = 1$ in Ω. Investigate if $f \in L^1(\Omega)$, $f \in L^1_{\mathrm{loc}}(\Omega)$ for $\Omega = (0,1)$ and $\Omega = \mathbb{R}$.

2. Let Ω be a bounded Lipschitz domain and let $u \in H^2(\Omega) \cap H_0^1(\Omega)$. Prove the interpolation inequality

$$\|\nabla u\|_{L^2(\Omega)}^2 \le \|u\|_{L^2(\Omega)} \|\Delta u\|_{L^2(\Omega)}.$$

3. Prove Lemma 3.41 (another Poincaré–Friedrichs inequality).

4. Prove Corollary 3.45: Let the conditions of Theorem 3.43 on the domain Ω be satisfied. Consider $u \in W^{1,p}(\Omega)$ and $v \in W^{1,q}(\Omega)$ with $p \in (1,\infty)$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then it is

$$\int_\Omega \partial_i u(\mathbf{x}) v(\mathbf{x}) \; d\mathbf{x} = \int_\Gamma u(\mathbf{s}) v(\mathbf{s}) \mathbf{n}_i(\mathbf{s}) \; d\mathbf{s} - \int_\Omega u(\mathbf{x}) \partial_i v(\mathbf{x}) \; d\mathbf{x}.$$

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Tuesday, May 28, 2013** either before or after one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.