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Numerical Mathematics III – Partial Differential Equations

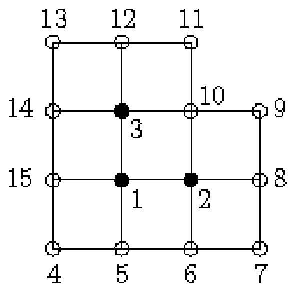
Exercise Problems 03

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Consider the Dirichlet problem for the Poisson equation

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega, \\ u &= g \quad \text{on } \partial\Omega, \end{aligned}$$

and the corresponding finite difference discretization with the five point stencil on the following grid:



Compute the matrices $A \in \mathbb{R}^{3\times 3}$ and $B \in \mathbb{R}^{3\times 12}$ for the finite difference equation

$$A\mathbf{u} = \mathbf{f} + B\mathbf{g}$$

with $\mathbf{u} = (u_1, u_2, u_3)^T$.

2. Prove the comparison lemma (Corollary 2.22).

3. **This problem has to be solved until May 07, 2013.** Write a MATLAB code for the numerical solution of

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega = (0,1)^2, \\ u &= g \quad \text{on } \partial\Omega. \end{aligned}$$

The right hand side and the boundary conditions should be chosen such that

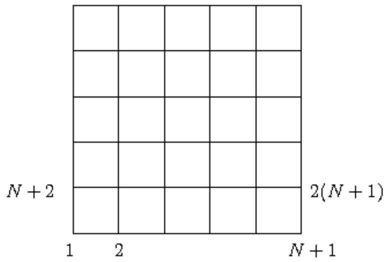
$$u(x,y) = x^4 y^5 - 17 \sin(xy)$$

is the solution of the boundary value problem.

Use the five point stencil for the discretizing the partial differential equation. The mesh widths should be chosen to be

$$h_x = h_y = h = 2^{-n} \quad n = 2, 3, 4, \ldots, 8.$$

Order the nodes lexicographically and store the matrix in `sparse` format.



Compute the following errors

$$\|u - u_h\|_{l^\infty(\omega_h)}, \qquad \|u - u_h\|_{l^2(\omega_h)},$$

and the orders of convergence, based on the errors on the two finest meshes, where the second norm is the standard Euclidean vector norm.

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Tuesday, Apr. 30, 2013** either before or after one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.