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Exercises to the classes
Numerical Methods in Sciences and Technics

Exercises no. 11
to 19.01.2004

The solution of exercise 2 is to submit in the exercise classes on Monday, 19.01.2004 !

Statements given in the lecture can be used in the solution of the exercises without proof. All other statements have to be proved.

1. Prove Gronwall's lemma: Let $\phi(t) \; : \; [t_0, t_e] \to \mathbb{R}$ be continuous with

$$\phi(t) \leq \alpha + \beta \int_{t_0}^{t} \phi(\xi) d\xi$$

   for all $t \in [t_0, t_e]$, $\alpha, \beta \in \mathbb{R}$, $\alpha, \beta > 0$. Then

$$\phi(t) \leq \alpha \exp(\beta(t - t_0)).$$

   Hint : use the result of

$$\frac{d}{dt} \ln \left(\alpha + \beta \int_{t_0}^{t} \phi(\xi) d\xi \right).$$

2. Compute the differentiation index of the DAE

$$\begin{aligned} 0 &= g(v) \\ v' &= f(v, w) \\ w' &= k(v, w, u) \end{aligned}$$

   where $g_v f_w k_u$ has a bounded inverse.