Lösungen zum 35. Präsenzblatt für MfI 3

1. Aufgabe :

(a)

$$\begin{aligned} \mathbf{f}(x,y) &= \begin{pmatrix} 12xy+3 \\ 6x^2 \end{pmatrix} \\ \nabla \times \mathbf{f}(x,y) &= (12x - 12x)\,\mathbf{e}_z \\ &= \mathbf{0} \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{f}(x,y) &= \begin{pmatrix} 3x^2y \\ x^3 \end{pmatrix} \\ \nabla \times \mathbf{f}(x,y) &= \left(3x^2 - 3x^2\right)\mathbf{e}_z \\ &= \mathbf{0} \end{aligned}$$

(c)

$$\begin{aligned} \mathbf{f}(x,y) &= \begin{pmatrix} x+z \\ -y-z \\ x-y \end{pmatrix} \\ \nabla \times \mathbf{f}(x,y) &= (-1-(-1))\,\mathbf{e}_x + (1-1)\,\mathbf{e}_y + (0-0)\,\mathbf{e}_z \\ &= \mathbf{0} \end{aligned}$$

2. Aufgabe :

$$\begin{aligned} \text{Jacobi-Matrix} : J_f &= \left(\frac{\partial f_i}{\partial x_j}\right)_{i=1,\ldots,m, j=1,\ldots,n} \\ &= \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \det \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial z} \end{pmatrix} &= \det \begin{pmatrix} \cos(\varphi) & -r\sin(\varphi) & 0 \\ \sin(\varphi) & r\cos(\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= r \end{aligned}$$