Phase transition for continuum Gibbs particle systems

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Abstract

In this mini-course we give a state of the art of the phase transition phenomenon for continuum interacting particles system. The model is defined in the infinite volume regime via the DLR equations which prescribe the local conditional densities following the Gibbs-Boltzmann formalism. The equilibrium states are also the thermodynamic limit of finite volume volume Gibbs models for any boundary condition. Changing the boundary condition at infinity can change drastically the equilibrium states. We call this phenomenon, phase transition. Two kind of phase transition can appear. The first one, called liquid-gas phase transition, preserves the symmetries of the model (translations, rotations, etc) but the density, the energy or the entropy of particles change abruptly at some values of parameters (in general activity or inverse temperature). The second one, called symmetry breaking phase transition, break some symmetries of the model even though the interaction satisfy these symmetries. Depending on the model, the translation or the rotation invariance is violated.

This topic has a long history in Physics and Mathematics-Physics. Several conjectures have been claimed more than fifty years ago and most of them are still open today. Only few results have been proved rigorously due to the lack of tractability of models. Moreover, in the continuum setting, the combinatorial tools, largely used for Gibbs models with bounded spins, is not really efficient here. However, for some models with well-chosen interactions, it is possible to prove the phase transition phenomena mentioned above. The proofs are rigorous and are based on several tools in Analysis, Probability theory, Geometry. During the mini-course, we will present these results, we will give partially the proofs and we will present also a large collection of conjectures provided beautiful challenges for present and probably future generations.

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