Hot Wire Cuttings for the Building Industries

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*Ph.D. students, i.e., doing the work.

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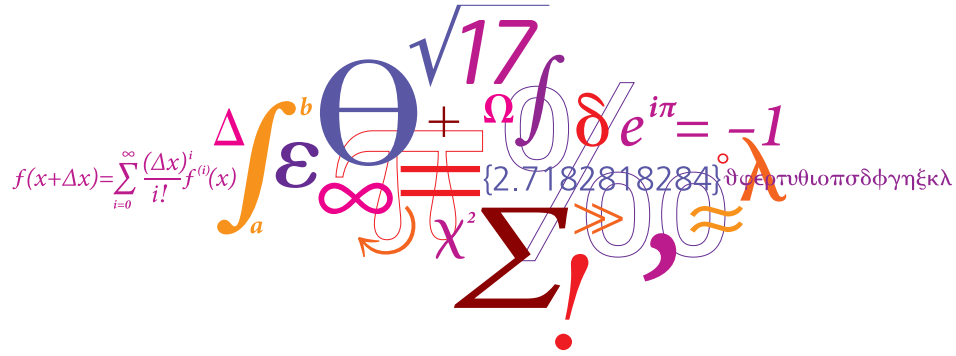
Technical University of Denmark

MaDiFa
Weierstrass Institute, Berlin, 2014



BladeRunner Project 2013–2016

Three year project 2013–2016 supported with 1.5 million EURO by *The Danish National Advanced Technology Foundation*

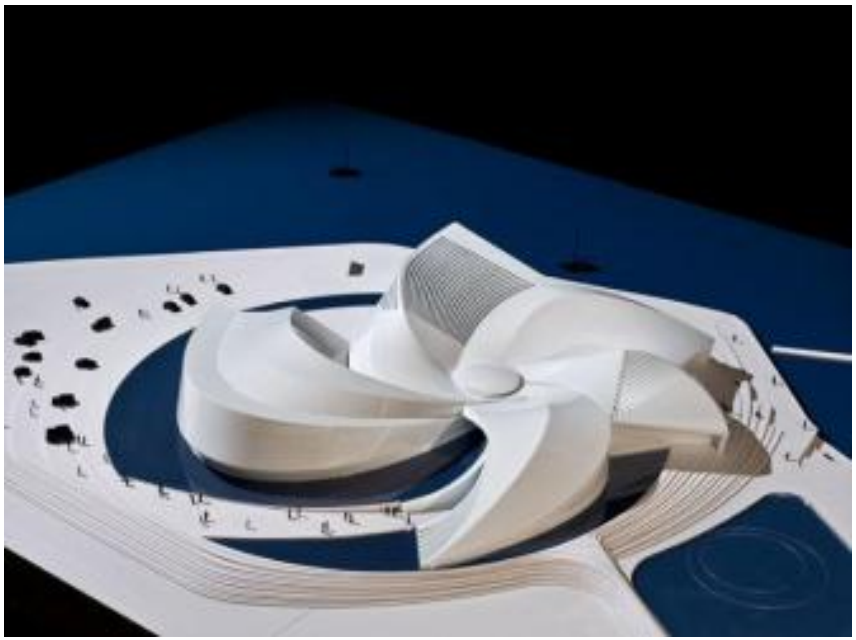
Partners:

- Odico: Project manager
  HotWire and HotBlade technology
- CONFAC:
  Concrete elements
- 3XN:
  Architectural design
- Danish Technological Institute:
  Robot technology
- The Technical University of Denmark:
  DTU Compute: Surfaces and rationalization (2 Ph.D. students)
  DTU Mechanical Engineering: Thermal and mechanical modeling (1 Ph.D. student)

Outline

Is the building industry an industry?

- Labor takes ∼50% of the cost in the Danish building industry
- The corresponding number is 10–20% for other industries in Denmark
- Architecture with non-trivial geometry is even more labor intensive



The architecture of "standard" buildings is mostly very boring.

3XN architects

Lighthouse, Aarhus, Denmark.

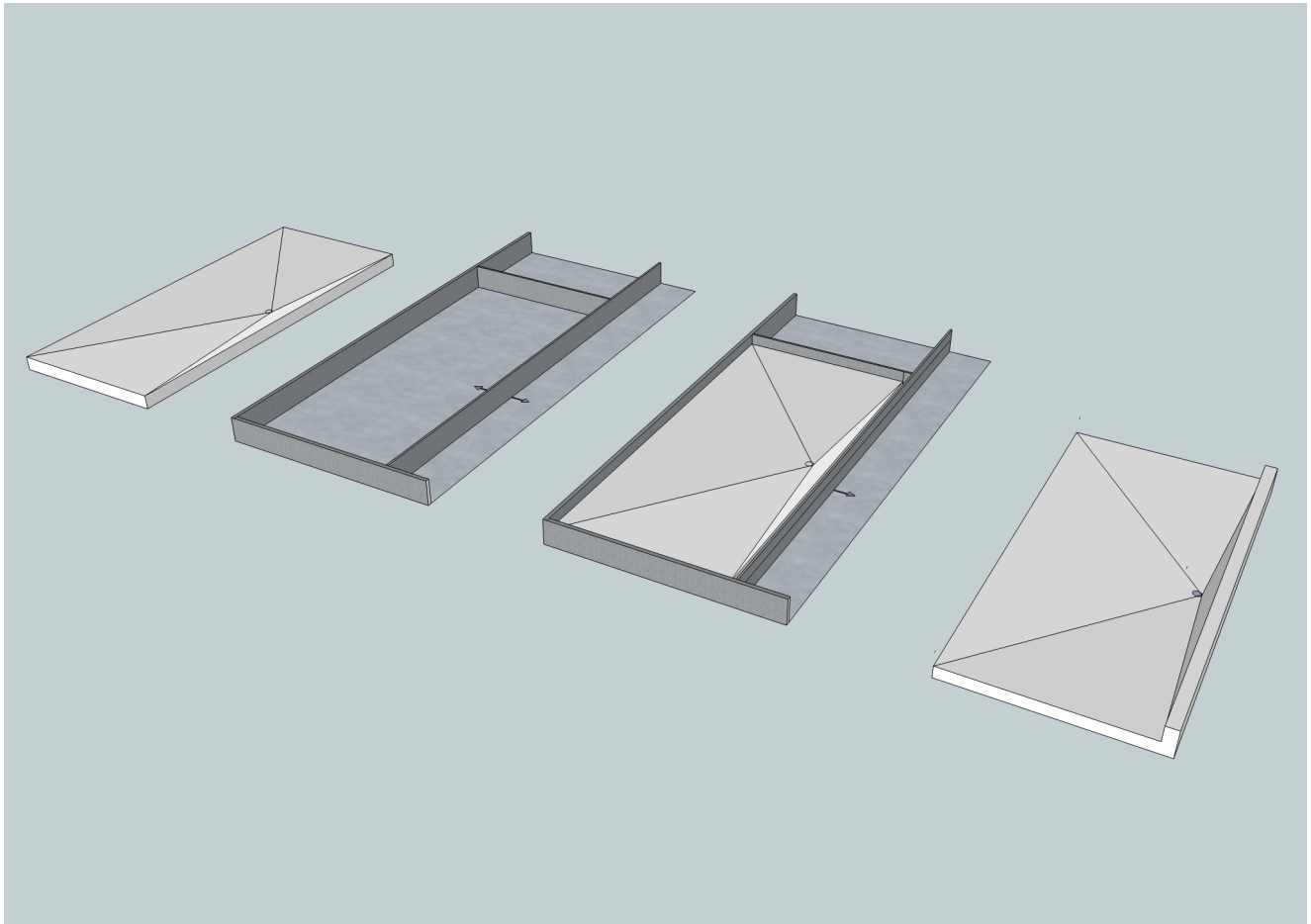
3XN architects

Blue Planet, Copenhagen, Denmark.
Design for mixed-use high rise building.

Handmade construction of a mold (wooden formwork).

CONFAC: Handmade unique plywood-mold for a balcony

CONFAC: EPS-form work directly on casting-bed (table)

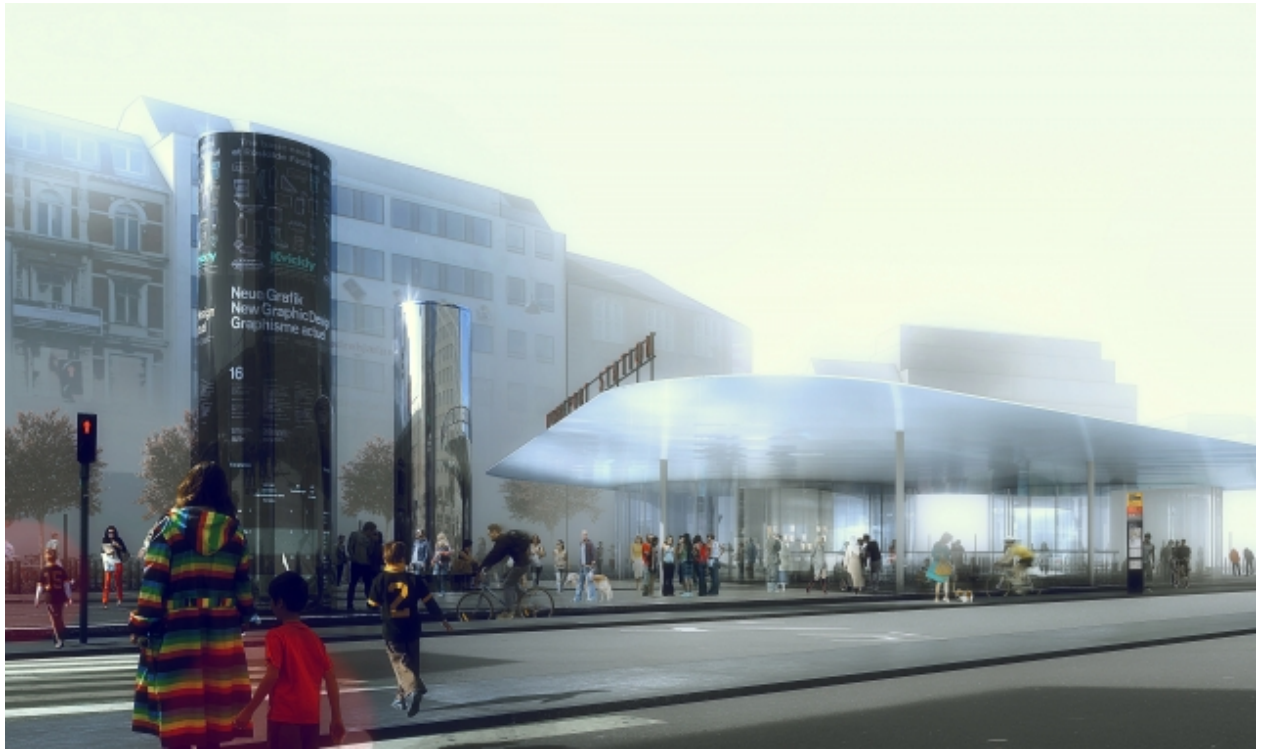


The price is reduced to the half.

Snøhetta: San Fransisco Museum of Modern Art

The EPS-mold for the facade requires one year of non stop NC milling.
$\rightarrow$ estimated 2–3 weeks of HotWire cutting

Concepts and ideas : An example



Concepts and ideas : An example

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Concepts and ideas : An example

It is :

1. Ad hoc
2. Imprecise
3. Messy, dirty
4. Timeconsuming
5. and very Costly

Robotic Hot Wire Cutting

Robotic Hot Wire Cutting

Robotic Hot Wire Cutting

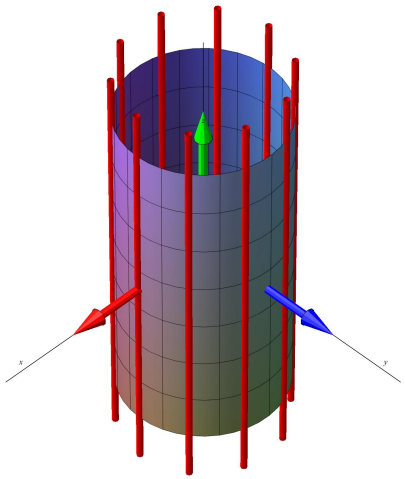
Robotic Hot Wire Cutting

It is :

1. Automatic
2. Precise
3. Clean (?)
4. Fast
5. and relatively Cheap

Robotic Hot Wire Cutting

But it can only produce surfaces with
non-positive Gaussian curvature :



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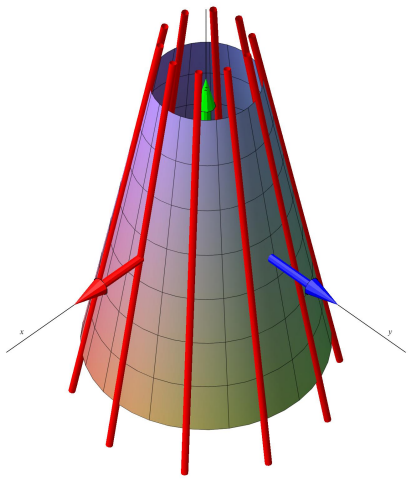

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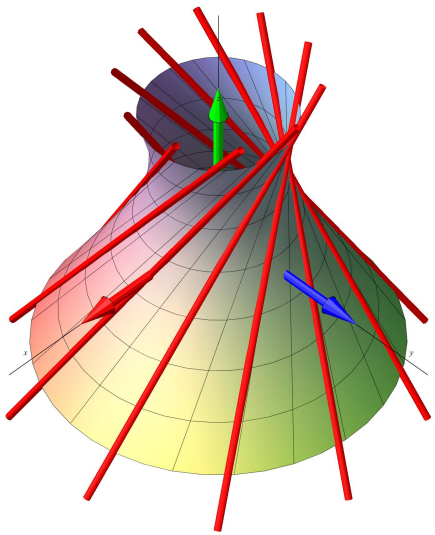

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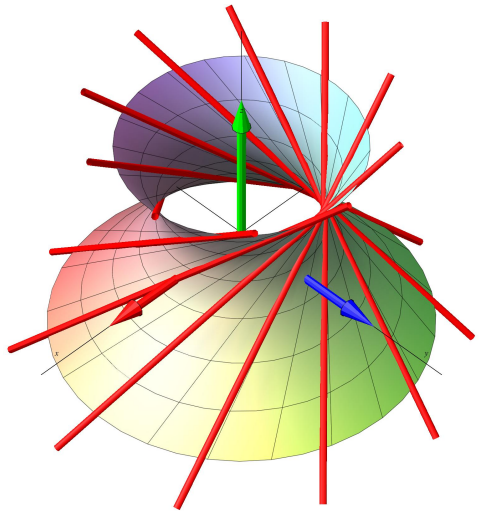

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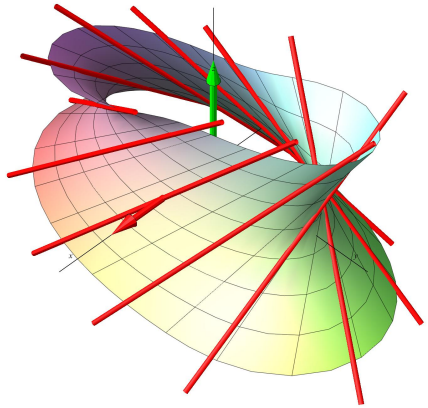

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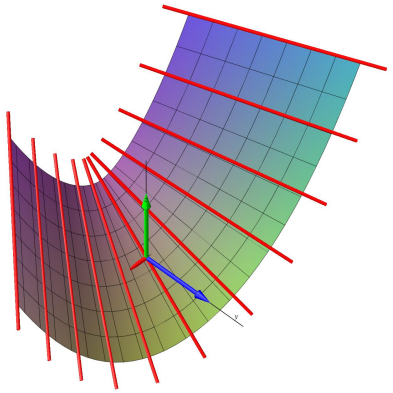

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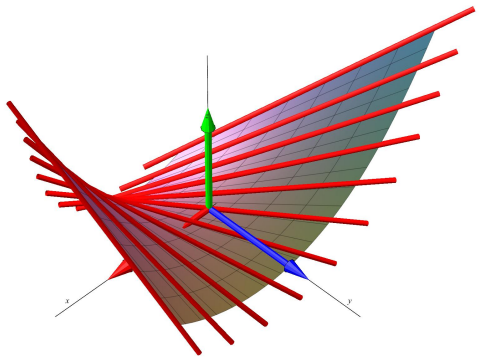

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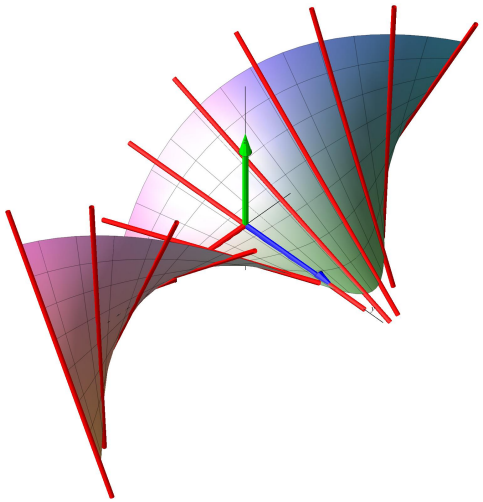

Robotic Flexible Blade Cutting a´la Euler

Positive Gaussian curvature can be obtained
by sweeping ELASTIC CURVES :

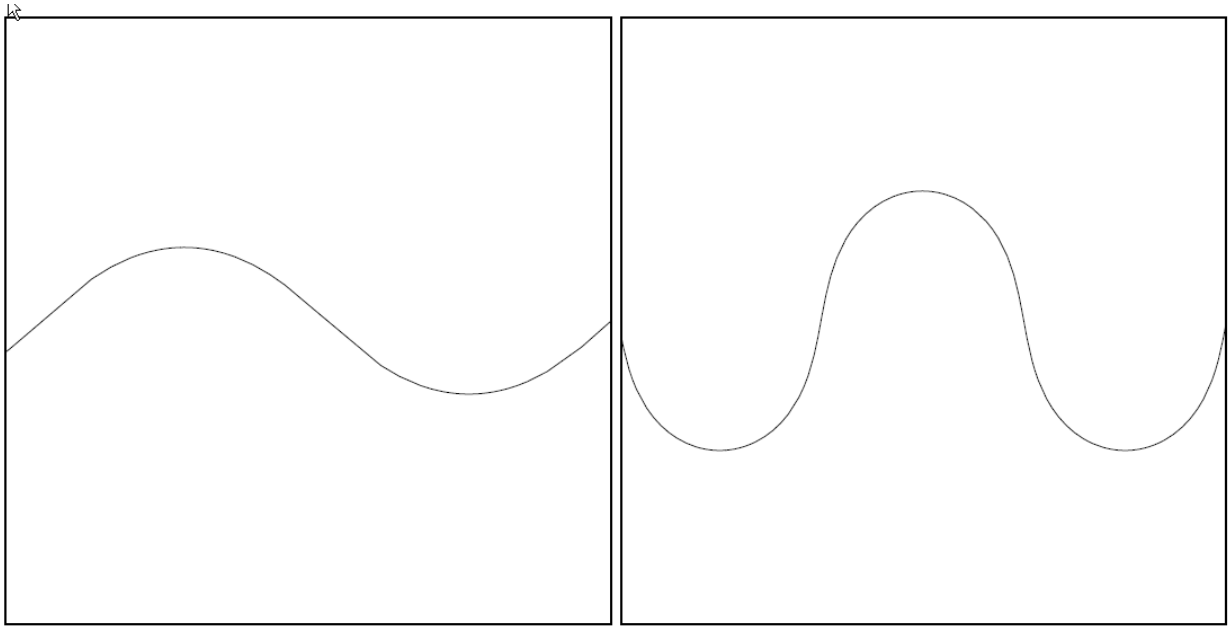

FIGURE 1. Wavelike elastica

$$\gamma(s) = \left(-2k\,\mathrm{cn}(s)\,,\ s - 2\,\mathrm{E}(\mathrm{am}(s,k),k)\right) \tag{1}$$

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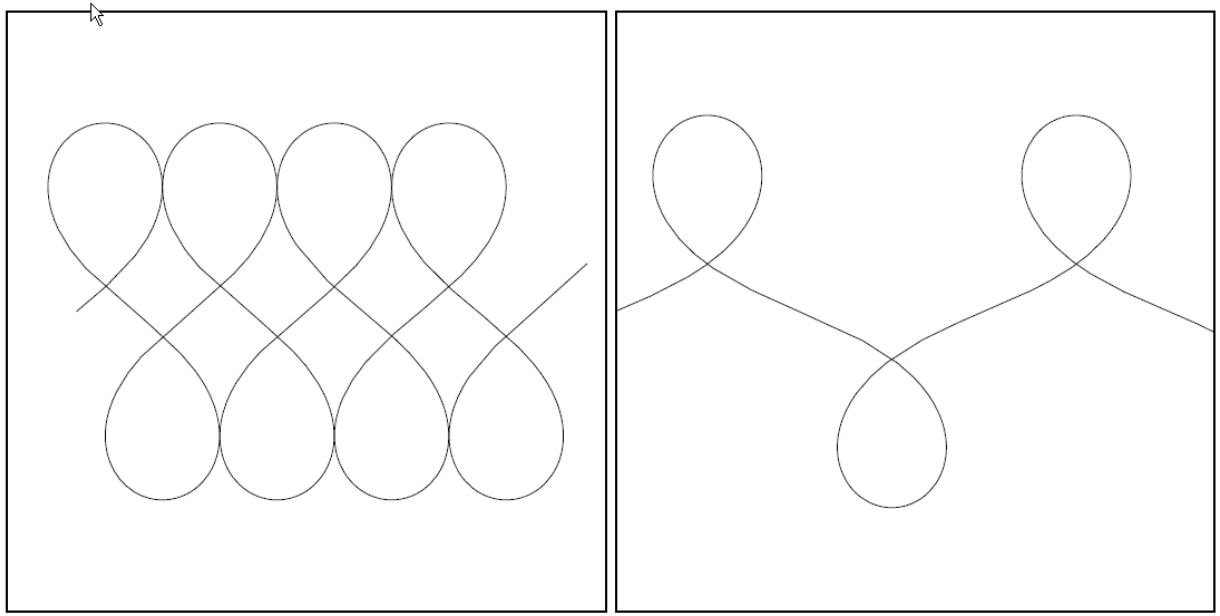

FIGURE 2. Wavelike elastica

$$\gamma(s) = \left(-2k\,\mathrm{cn}(s)\,,\ s - 2\,\mathrm{E}(\mathrm{am}(s,k),k)\right) \tag{2}$$

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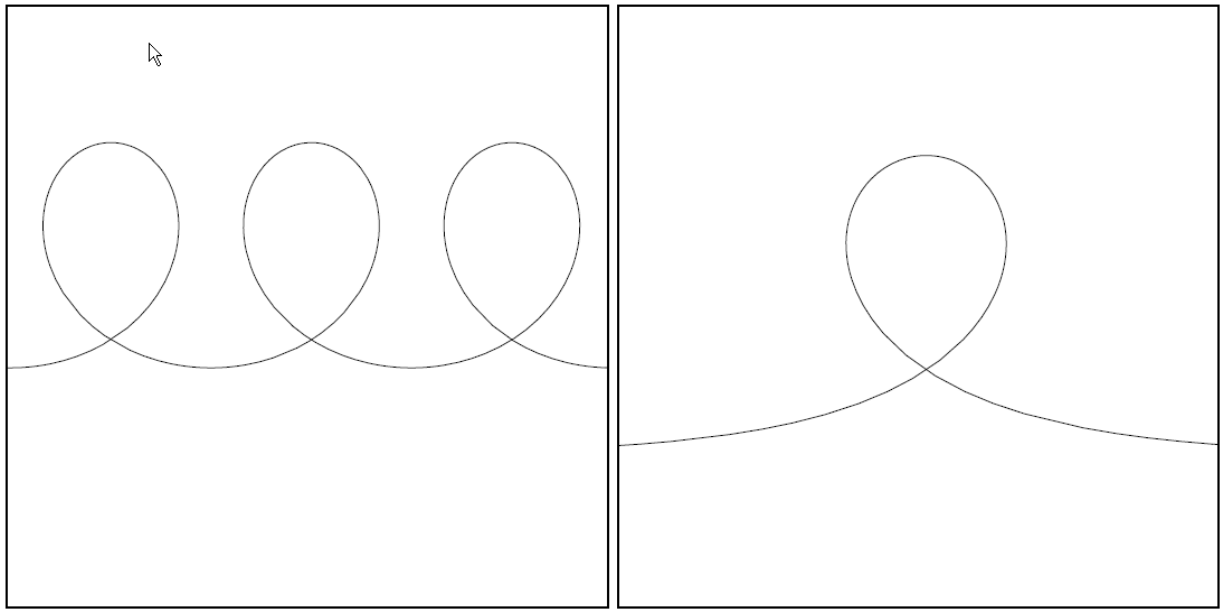

FIGURE 3. a) Orbitlike elastica b) Borderline elastica

$$\gamma(s) = \left(-\frac{2}{k}\,\mathrm{dn}\left(\frac{s}{k}\right),\ s\left(\frac{2}{k^2} - 1\right) - \frac{2}{k}\,\mathrm{E}\left(\mathrm{am}\left(\frac{s}{k}, k\right), k\right)\right) \tag{3}$$

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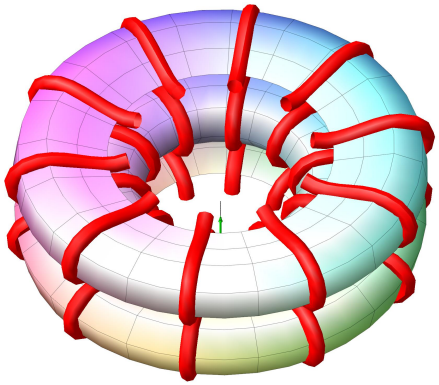

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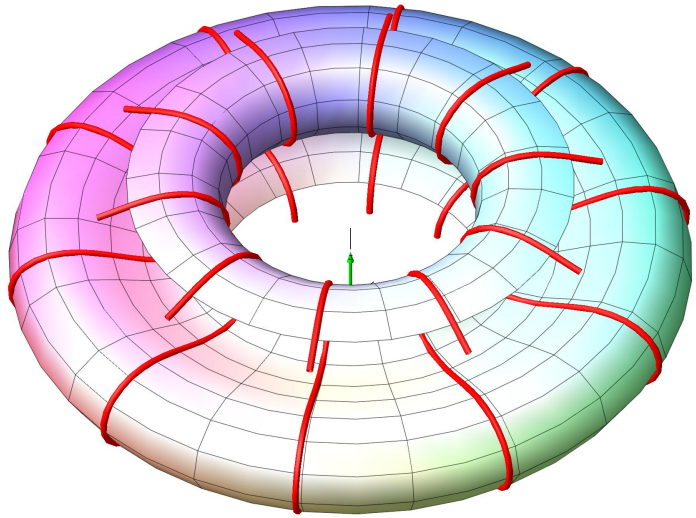

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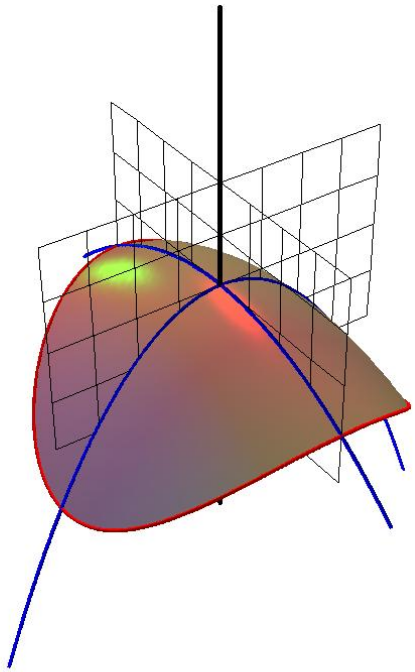

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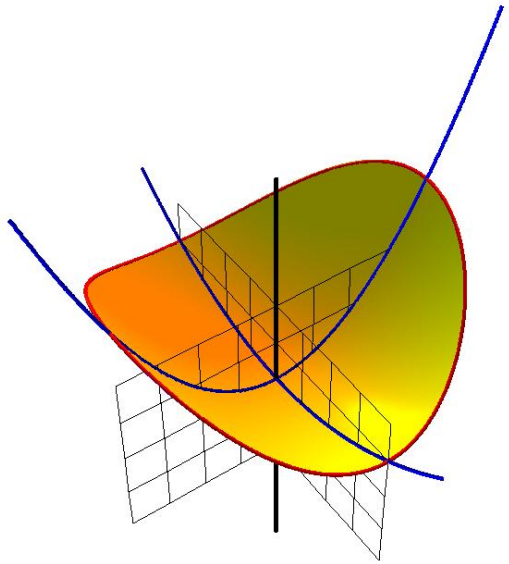

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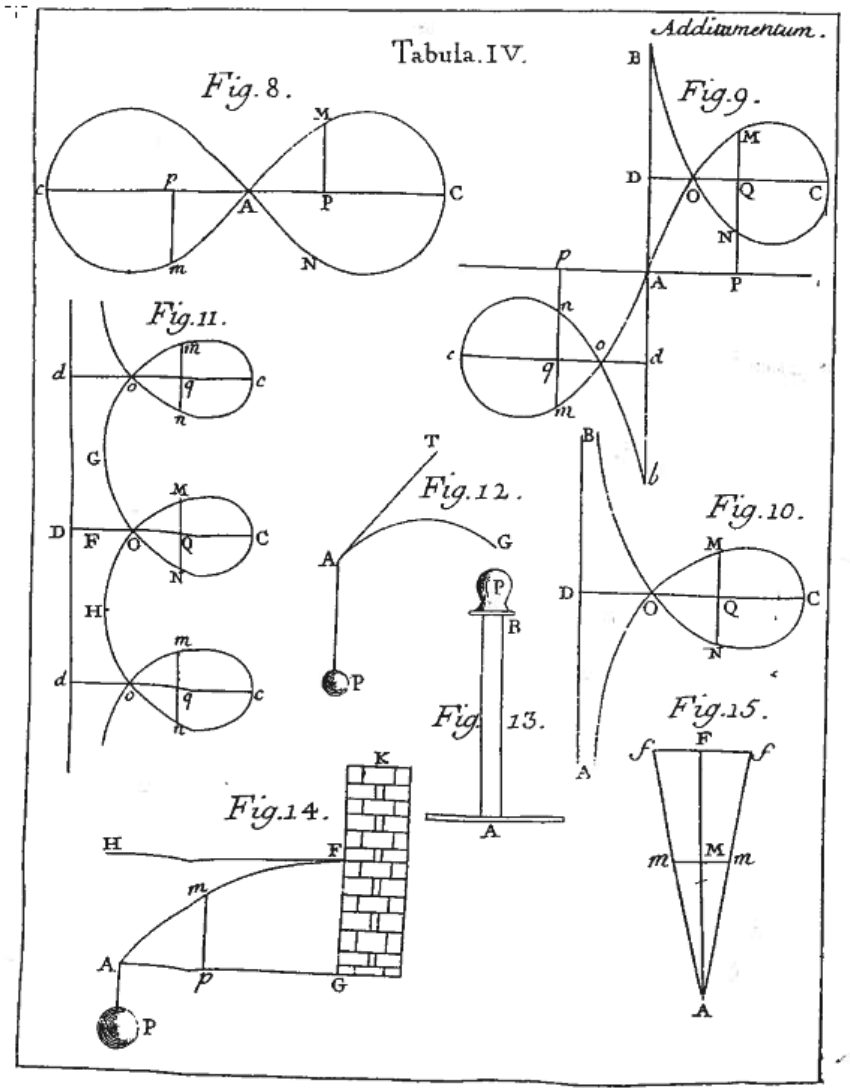
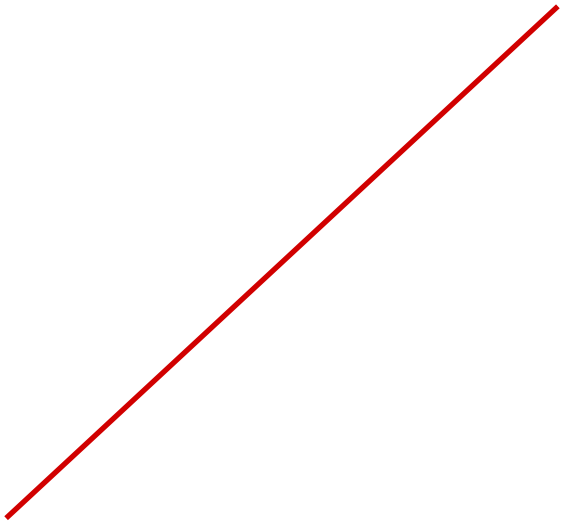
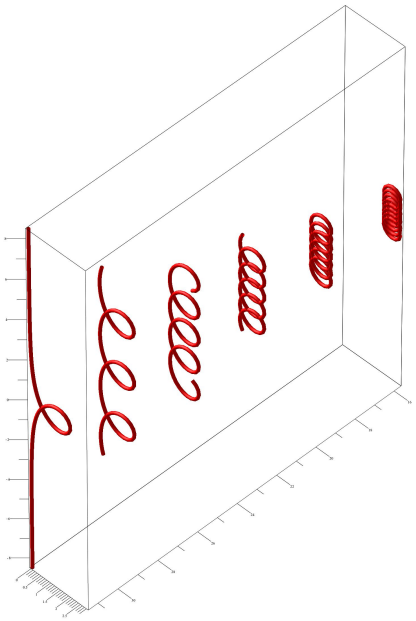


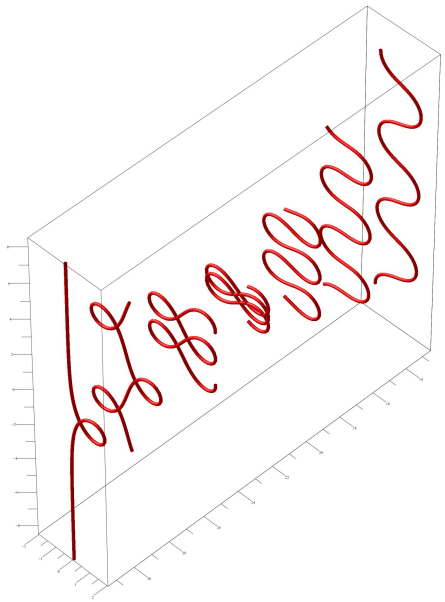
Robotic Flexible Blade Cutting a´la Euler

The HotWire and HotBlade technology

- Cutting with a wire $\rightarrow$ ruled surface (or is it?)
- Cutting with an elastic blade $\rightarrow$ surface swept by elastica (or is it?)







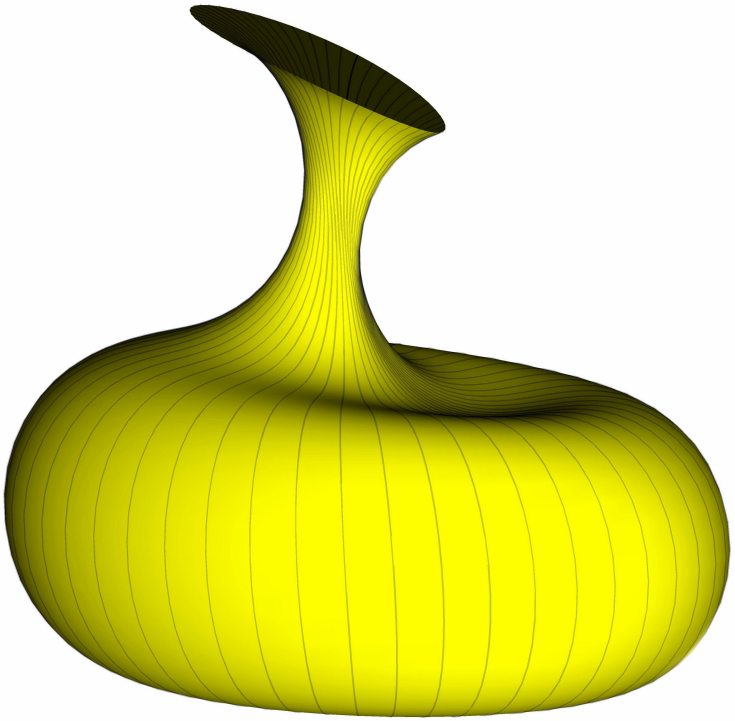
The "or is it" question will be answered by DTU Mechanical Engineering and by experiments.

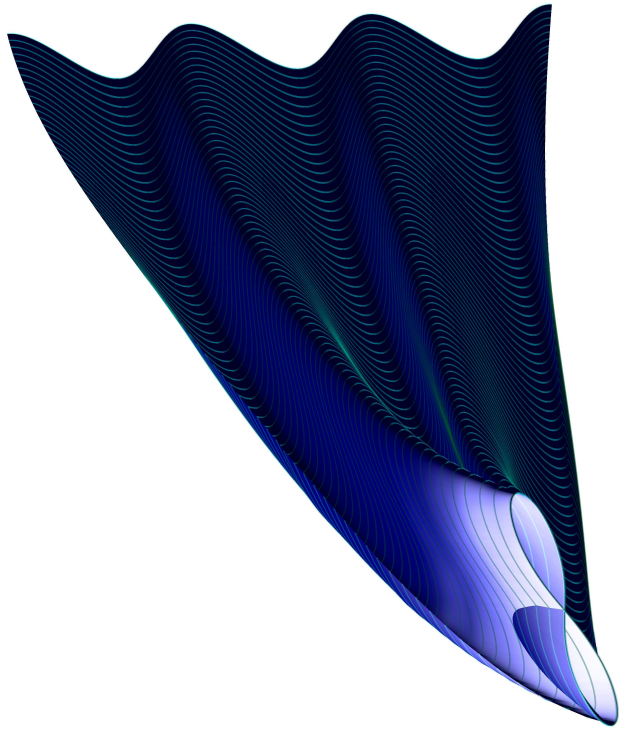
The mathematical problem: Surface rationalization

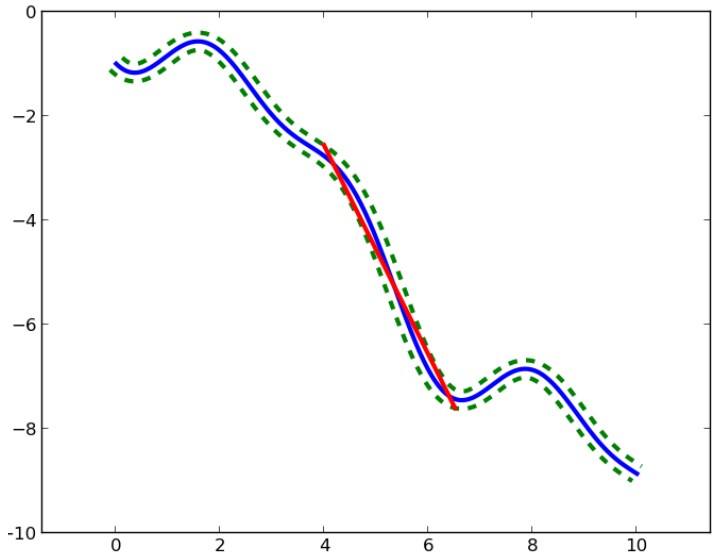
- Given a CAD model of the architecture or more precisely a spline surface modeling some ("interesting/expensive") piece of the building.
- Approximate it by a collection of surfaces that can be produced by the HotWire or the HotBlade technology.
- That is, approximate it by a collection of ruled surface and surfaces swept by elastica.

Surfaces swept by elastica

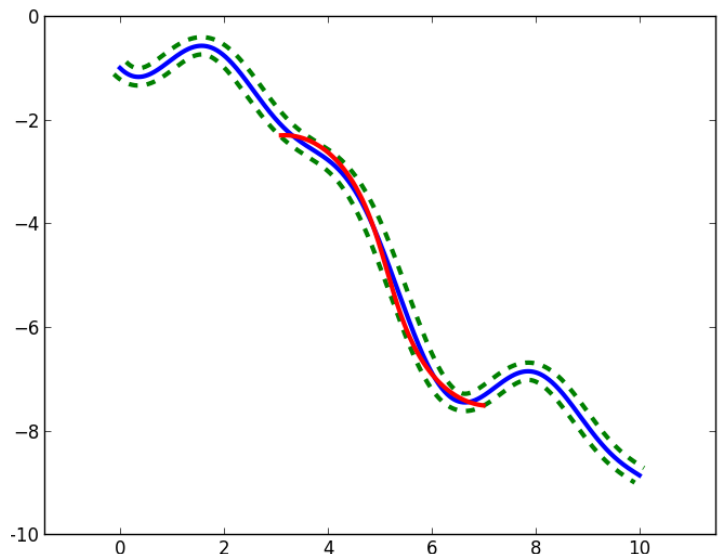
- Not studied before (to our knowledge)
- Can produce surfaces with arbitrary curvature
- Can be pieced together with C^2 continuity
- An elasticum can be expressed in closed form using elliptic functions



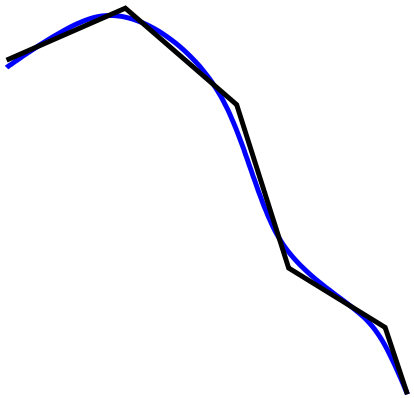




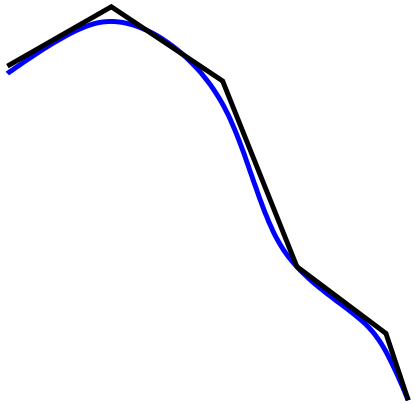
Approximation by lines.



Approximation by elastica.



Approximation.



One sided approximation.

The one sided approximation can be the starting point for NC-milling.

Other problems

- Smoothness conditions/tolerances between patches
- Distance measures
- Segmentation adapted to the block structure
- Production constraints
- Elastic and thermal deformations

Conclusion

- There is a need for industrialization of the building industry
- The BladeRunner project is a contribution to this
- Mathematics, including differential geometry, has a key role to play in this endeavor

Thank you for your attention!