Existence and approximation of a viscoelastic Cahn–Hilliard model for tumour growth

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In this talk, we consider a macroscopic model for tumour growth in which cell-cell adhesion effects are taken into account with the help of a Ginzburg–Landau type energy. The resulting evolution is given by a Cahn-Hilliard equation with source and sink terms. Moreover, nutrient diffusion is included by a coupling to a reaction-diffusion equation. The system is coupled to an internal non-solenoidal velocity field which solves a viscoelastic system. It is well-known that the invasive potential of tumours can be enhanced by mechanical forces, as in particular the movement and growth of tumour cell aggregates can be directed by mechanical effects. Moreover, a lack of nutrients can favour chemotaxis which in general describes the movement along the gradient of a chemical species like nutrients. To include both phenomena in our model, we consider specific energy densities that are coupled to the phase-field variable, which results in cross-diffusion terms in the Cahn–Hilliard equation.

For the full model, we present the numerical approximation with a fully-practical, stable and converging finite element scheme in two and three space dimensions, which preserves the physical properties of the model. Here, we introduce new approximations of some specific terms in the system with the goal to mimic an energy identity on the fully-discrete level. As the discretization parameters tend to zero, we pass to the limit in the numerical scheme and show (subsequence) convergence towards a global-in-time weak solution in two and three space dimensions. Finally, we illustrate the practicability of the discrete scheme with the help of numerical simulations.

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