On a Diffuse Interface Model for Incompressible Viscoelastic Two-Phase Flows

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1. Abstract

This talk concerns a diffuse interface model for the flow of two incompressible viscoelastic fluids in a bounded domain. More specifically, the fluids are assumed to be macroscopically immiscible, but with a small transition region, where the two components are partially mixed. Considering the elasticity of both components, one ends up with a coupled Oldroyd-B/Cahn–Hilliard type system, which describes the behavior of two-phase viscoelastic fluids. Based on the total energy

$$\mathcal{E} = \underbrace{\int_\Omega \frac{\rho(\phi)}{2} |\mathbf{u}|^2 \, \mathrm{d}x}_{\text{Kinetic energy}} + \underbrace{\int_\Omega \frac{\mu(\phi)}{2} \mathrm{Tr}(\mathbb{B} - \ln \mathbb{B} - \mathbb{I}) \, \mathrm{d}x}_{\text{Elastic energy}} + \underbrace{\int_\Omega \tilde{\sigma} \Big(\frac{\epsilon}{2} |\nabla \phi|^2 + \frac{1}{\epsilon} W(\phi) \Big) \, \mathrm{d}x}_{\text{Ginzburg–Landau free energy}},$$

we derive a thermodynamically consistent model for two-phase incompressible viscoelastic fluids of Oldroyd-B type [3], which in particular cases describe a fluid-structure interaction problem as in [4]. This model provides a viscoelastic fluid counterpart of the celebrated Abels–Garcke–Grün (AGG) model [1] for incompressible two-phase viscous flows.

We prove the existence of weak solutions to the system in two dimensions for general (unmatched) mass densities, variable viscosities, different shear moduli, and a class of physically relevant and singular potentials that guarantee the order parameter stays in the physically reasonable interval. The proof is initiated by [2], which solved the AGG model by a time discretization scheme. We first introduce a novel spatial regularization of the original system, which preserves the energy-dissipation structure of the problem, as well as the uniform estimates. To address the regularized system, we propose a new *hybrid implicit time discretization* based on [2], which is solved by a Leray–Schauder argument, on noting the well-posedness of regularized Oldroyd-B equation for $\mathbb{B}$.

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References

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