On the singular limit problem for a class of first order nonlocal lane-changing traffic flow models

Felisia Angela Chiarello[(1)], **and Alexander Keimer**[(2)]

(1) University of L'Aquila, DISIM, Edificio Coppito 1,
Via Vetoio, 67100, Italy.
(2) Friedrich-Alexander-Universität, Department of Mathematics, Cauerstr. 11,
91058 Erlangen, Germany

e-mail: `felisiaangela.chiarello@univaq.it`

In this talk, we will present a nonlocal-to-local convergence result with applications to traffic models. In particular, we will consider a system of two (or more) nonlocal balance laws characterized by lane-changing functions on the right-hand sides and exponential kernels in the nonlocal terms of the flux functions, i.e.

$$\begin{aligned}
\partial_t \boldsymbol{\rho}^1(t,x) + \partial_x\Big(V_1(\mathcal{W}[\boldsymbol{\rho}^1](t,x))\boldsymbol{\rho}^1(t,x)\Big) &= S\big(\boldsymbol{\rho}(t,x), \mathcal{W}[\boldsymbol{\rho}](t,x), x\big) && (t,x)\in(0,T)\times\mathbb{R}\\
\partial_t \boldsymbol{\rho}^2(t,x) + \partial_x\Big(V_2(\mathcal{W}[\boldsymbol{\rho}^2](t,x))\boldsymbol{\rho}^2(t,x)\Big) &= -S\big(\boldsymbol{\rho}(t,x), \mathcal{W}[\boldsymbol{\rho}](t,x), x\big) && (t,x)\in(0,T)\times\mathbb{R}\\
\boldsymbol{\rho}(0,x) &= \boldsymbol{\rho}_0(x) && x\in\mathbb{R}\\
\mathcal{W}[\rho](t,x) &= \tfrac{1}{\eta}\int_x^\infty \exp\left(\tfrac{x-y}{\eta}\right)\rho(t,y)\,\mathrm{d}y && (t,x)\in(0,T)\times\mathbb{R}
\end{aligned}$$

with the nonlocal operator $\mathcal{W}$ defined for $\rho \in C([0,T]; L^1_{\text{loc}}(\mathbb{R})) \cap L^\infty((0,T); L^\infty(\mathbb{R}))$,

$$\mathcal{W}[\boldsymbol{\rho}](t,x) = (\mathcal{W}[\boldsymbol{\rho}^1], \mathcal{W}[\boldsymbol{\rho}^2])(t,x)$$

and $\boldsymbol{\rho} = (\boldsymbol{\rho}^1, \boldsymbol{\rho}^2)$. Here, the exponential kernels are approximations of the Dirac distribution and the coupling between the equations of the system is only through the right-hand sides. We will analytically prove that the solution of the nonlocal system converges to the solution of the corresponding local one when the kernels of the nonlocal terms approach the Dirac delta. Numerical illustrations supporting the main results will be also shown.

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References

[1] F. A. Chiarello and A. Keimer, On the singular limit problem for a class of first order nonlocal lane-changing traffic flow models. *In preparation.* (2023).