Equity derivatives pricing

With the equity derivatives market being a multiple of global GDP, the availability of efficient pricing and hedging techniques of derivatives (in particular European options) is essential for financial intermediaries around the world.

**Fundamental question:** How to model instantaneous stochastic volatility consistently across all strikes and maturities?

- Model needs to reproduce empirical implied volatility surface (Fig. 1).
- An important proxy for the surface is given by the term structure of the at-the-money (ATM) volatility skew (Fig. 2).

Two common problems

Diffusive stochastic volatility models commonly have two deficiencies:

- They lack a good agreement with market data under both the physical as well as the pricing measure.
- They fail to reproduce the power-law behaviour of the volatility skew for small times to maturity (Fig. 2).

**Revelation:** Volatility is rough

There is strong empirical evidence that log realized volatility has Hölder regularity much less than 1/2 (typically around 0.1). This is a ubiquitous phenomenon observed across thousands of equities and indices [GJR14, BLP16]. Hence, model log-volatility as a fractional Brownian motion with Hurst parameter \( H < 1/2 \).

The rough Bergomi model

For \( \mathcal{S} \), the S&P 500 Index, \((Z, W)\) a two-dim. BM with \( d(Z,W) = \rho \in (-1,1) \) and Riemann-Liouville fBM \( \hat{W} \) given by \( \hat{W} = \sqrt{2T} \int_0^t (t-s)^{H-1/2} dW_s \), consider

\[
dS_t = \sqrt{\sigma} S_t dZ_t, \quad \sigma_t = \xi_t \mathcal{E} \left( \eta \hat{W}_t \right)
\]

**Non-Markovianity resolved**

The instantaneous variance \( \sigma_t \) inherits non-Markovianity from fBM:

\[
\xi(u) = \mathbb{E} \left[ \sigma_t \bigg| \mathcal{F}_t \right] \neq \mathbb{E} \left[ \sigma_u \right]
\]

for \( u > t \) which is very bad from a simulation point of view. Fortunately, \( \xi(u) \) may be observed in the market (via variance swaps).

Simulation results

Rough Bergomi achieves a fantastic fit to real data! It captures

- the power-law behaviour of the SPX vol skew near zero (Fig. 2)
- the geometry of SPX smiles (Fig. 3)
- the SPX ATM implied volatility term structure

with only three parameters: the Hurst parameter \( H \) of the fBM, vol of vol \( \eta \) and correlation parameter \( \rho \) governing the leverage effect!

Two ongoing projects in rough vol framework

- Development of small-time asymptotic formulae for the implied volatility term structure etc. (with A. Gulisashvili (Ohio) and B. Horvath (Imperial))
- A novel pricing algorithm via the theory of regularity structures (with P. Gassiat (Dauphine) and J. Martin (Berlin))

References


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