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Symmetric local absorbing boundary conditions for the Helmholtz equation

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Local boundary conditions are used to mimick the solution in presence of an infinite exterior in diffusion problems or time-harmonic scattering problems, in highly conducting bodies or thin layers. We consider symmetric local boundary conditions in $\mathbb{R}^2$, which take the following form of Dirichlet-to-Neumann maps [1, Eq. 3.14]

$$\partial_n u(x) + \sum_{j=0}^{J} (-1)^j a_j \, (\partial_\tau^{2j} u)(x) = 0, \tag{1}$$

where ∂_n and ∂_τ are the normal and tangential derivatives on the boundary of the computational domain, respectively, and a_j are some complex-valued coefficients. The parameter J corresponds to the order of the derivatives. If $J < 0$ then (1) is known as the (homogeneous) Neumann boundary conditon, for $J = 0$ and $J = 1$ as Robin or Wentzell boundary condition, respectively. Prominent examples of symmetric local boundary conditions are the Bayliss-Turkel-Gunzberger (BGT) conditions up to order 2 and Feng's conditions at any order for time-harmonic scattering problems. Here, the problem is restricted to a computational domain of radius R with boundary conditions on its "artificial" boundary.

The talk deals with the analysis of the Helmholtz equation with symmetric local absorbing boundary conditions, *i.e.*, well-posedness and existence of (spurious) eigenmodes. We will show exact, but non-local Dirichlet-to-Neumann maps for scattering problems and show a derivation of Feng's conditions as an approximation in form of (1) based on asymptotic behaviour of the Hanckel functions for large arguments. The estimation of the modelling error in terms of R has to face the problem that the size of the domain depends on R. We aim to give some ideas of a rigorous error analysis. The sharpness of the error bounds are verified using numerical experiments based on an interior penalty finite element method.

References

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