Energy estimates, relaxation, and existence for strain gradient plasticity with cross hardening

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We consider a variational formulation of gradient elasto-plasticity subject to a class of single-slip side conditions. Such side conditions typically render the associated boundary-value problems non-convex. We first show that, for a large class of plastic deformations, a given single-slip condition (specification of Burgers' vectors and slip planes) can be relaxed by introducing a microstructure through a two-stage process of mollification and lamination. This yields a relaxed side condition which only prescribes slip planes and allows for arbitrary slip directions. This relaxed model can be thought of as an aid to simulating macroscopic plastic behavior without the need to resolve arbitrarily fine spatial scales. We then discuss issues of existence of solutions for the relaxed model.

Finally, we apply this relaxed model to a specific system, in order to be able to compare the analytical results with experiments. A rectangular shear sample is clamped at each end, and is subjected to a prescribed horizontal, modelled by an appropriate Dirichlet condition. We ask: how much energy is required to impose such a shear, and how does the energy depend on the aspect ratio of the sample? Assuming that just two slip systems are active, we show that there is a critical aspect ratio, above which the energy is strictly positive, and below which it is zero. Furthermore, in the respective regimes determined by the aspect ratio, we prove energy scaling bounds, expressed in terms of the amount of prescribed shear.