L^p-extrapolation of the generalized Stokes operator

Patrick Tolksdorf

Johannes Gutenberg-Universität Mainz
Institut für Mathematik
Staudingerweg 9
55128 Mainz
Germany
tolksdorf@uni-mainz.de

In this talk, we discuss the Stokes operator with bounded measurable coefficients μ, formally given by

$$Au := -\mathrm{div}(\mu \nabla u) + \nabla \phi, \quad \mathrm{div}(u) = 0 \quad \text{in } \mathbb{R}^d. \tag{1}$$

As this operator arises as a linearization of non-Newtonian fluids, optimal regularity estimates are of particular importance. Under mild ellipticity assumptions on μ, standard form methods show for example, that A satisfies L^2-resolvent estimates of the form

$$\|\lambda(\lambda + A)^{-1} f\|_{\mathrm{L}^2} \leq C \|f\|_{\mathrm{L}^2} \qquad (f \in \mathrm{L}^2_\sigma(\mathbb{R}^d))$$

for λ in some complex sector $\{z \in \mathbb{C} \setminus \{0\} : |\arg(z)| < \theta\}$, for some $\theta > \pi/2$, and thus $-A$ generates a bounded analytic semigroup e^{-tA} on L^2_σ. We describe how an analogue of such a resolvent estimate can be established in L^p by virtue of certain non-local Caccioppoli inequalities combined with an extrapolation argument of Shen. Such estimates build the foundation for many important functional analytic properties of these operators like maximal L^q-regularity and the boundedness of its H^∞-calculus.

More precisely, we establish resolvent estimates in L^p for p satisfying

$$\Big| \frac{1}{p} - \frac{1}{2} \Big| < \frac{1}{d}. \tag{2}$$

This resembles a well-known situation for elliptic systems in divergence form with L^∞-coefficients. Here, important estimates like Gaussian upper bounds for the semigroup cease to exist and the L^p-extrapolation has be concluded by other means. In particular, for elliptic systems one can establish resolvent bounds for numbers p that satisfy (2). Moreover, if $d \geq 3$, Davies constructed examples which show that corresponding resolvent bounds do not generally hold in L^p for numbers $1 < p < \infty$ that satisfy

$$\Big| \frac{1}{p} - \frac{1}{2} \Big| > \frac{1}{d}.$$

These elliptic results give an indication that the corresponding result for the Stokes operator with L^∞-coefficients is optimal.