Breakdown of the mean-field description of interacting systems: Phase transitions, metastability and coarsening

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We present results concerning the qualitative and quantitative description of interacting systems, with particular emphasis on those possessing a phase transition under the change of relevant system parameters.

For this, we first discuss and identify continuous and discontinuous phase for mean-field limits of interacting particle systems on the torus and spheres.

Since phase transitions are intimately related to the metastability of the stochastic particle system, we show how a suitable mountain pass theorem in the space of probability measures can describe the metastable behaviour of the underlying finite particle system.

We also argue that the mean-field description of the particle system in the regime of strong local interaction has to break down. In this regime, coarsening is observed, where smaller clusters grow through coagulation events. We provide numerical experiments with a positivity preserving numerical scheme for a SPDE of Dean-Kawasaki type, consisting of the McKean-Vlasov equation and conservative noise.

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