Gradient flows on metric graphs with reservoirs

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We study an evolution equation on metric graphs with reservoirs, that is graphs where a one-dimensional interval is associated to each edge and, in addition, the vertices are able to store mass. We focus on the case then the dynamics is driven by an entropy functional, defined both on edges and vertices. We provide a rigorous understanding of such equations as a gradient flow (in continuity equation format) with respect to metric that allows for a coupling between edge and vertex dynamics. By approximating the edges by a sequence of vertices, resulting in a fully discrete system, we are able to establish existence of solutions in this formalism. Next, we study several scaling limits and using in the framework of EDP convergence with embedding we are able to rigorously show convergence to again gradient flows on reduced graphs. Finally, numerical studies confirm our theoretical findings.