

A simple approach to the chain rule for gradient systems

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For constructing solutions for gradient systems, one usually approximates the problem, e.g. via the minimizing movement scheme, and then passes to the limit. By this way one obtains an energy-dissipation inequality and has to prove that this inequality is indeed an equality. For this one has to establish a chain rule for the energy functional within the class of curves satisfying only the natural dissipation and slope bounds.

We present a simple approach that is based on the classical duality pairing in L^p spaces instead of the often used metric approach in measure spaces. We show that this approach allows the treatment of a reasonably large class of nonlinear diffusion equations. Moreover, it is flexible enough to be generalized to systems of diffusion equations.