Covariance modulated optimal transport: geometry and gradient flows

Daniel Matthes (TU München)

This talk is about a novel variant of optimal mass transport in which particles move with a global linear but anisotropic mobility depending on the covariance of the ensemble density. This gives rise to a novel kind of Wasserstein metric that provides a rigorous gradient flow formulation of the mean-field limit for the ensemble Kalman sampling. In combination with the abstract machinery of metric gradient flows, the new metric is an effective tool to study the rate of convergence of these methods.

I shall present several analytic results about the modulated Wasserstein metric. The first is the splitting representation, which allows to write the modulated metric as the sum of two simpler metrics, one measuring the distance in terms of first and second moments, the other one measuring in terms of shapes. The second result is about geodesic convexity and the related rates of convergence in gradient flows. Specifically, we prove exponential equilibration in linear Fokker-Planck equations with Gaussian steady states at a rate that does not depend on the covariance of the Gaussian. The third and only partial result is about geodesics, that we prove to exist for sufficiently close densities, or densities with multiple reflection symmetries. We also characterize geodesics in terms of particle trajectories, that are no longer straight line as in the genuine Wasserstein metric, but follow more complicated curves that satisfy second order ordinary differential equations.

This is joint work with Andre Schlichting, Matthias Erbar, Franca Hoffmann and Martin Burger.