The Cahn-Hilliard Equation with Dynamic Boundary Conditions and its Application to Two-Phase Flows

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The Cahn-Hilliard equation is the most common model to describe phase separation processes in a mixture of two materials. Moreover, it is further used to describe different phenomena where the distribution and/or motion of two (or more) immiscible materials is considered.

Standard Cahn-Hilliard models are usually endowed with homogeneous Neumann boundary conditions for both the phase-field variable and the chemical potential. However, these boundary conditions yield certain limitations:
1.) The diffuse interface separating the materials is enforced to intersect the boundary at a perfect angle of ninety degrees, which is unrealistic in many applications.
2.) No transfer of material between bulk and boundary is allowed and thus, absorption process cannot be described. For these reasons dynamic boundary conditions for the Cahn-Hilliard equation have been introduced. We take a closer look at dynamic boundary conditions that also exhibit a Cahn-Hilliard type structure.

To describe the evolution of two-phase flows, Navier-Stokes-Cahn-Hilliard models have become a popular choice. As the standard models are subject to a no-slip boundary condition for the velocity field as well as homogeneous Neumann conditions for the Cahn-Hilliard subsystem, they exhibit the aforementioned limitations and are also not well-suited for describing general moving contact line phenomena. However, these issues can also be overcome by the introduction of suitable dynamic boundary conditions.