The three limits of the hydrostatic approximation

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The primitive equations are a large scale model for ocean and atmosphere. Formally, they are derived from the 3D-Navier-Stokes equations by the assumption of a hydrostatic balance. This can be formalized by a rescaling procedure on an ε-thin domain where one considers anisotropic viscosities with vertical viscosity ε^γ and ε-independent horizontal viscosity. Now, the choice of the order γ leads to different limit equations:

- For $\gamma = 2$, one obtains the primitive equations with full viscosity term $-\Delta$;
- For $\gamma > 2$, one obtains the primitive equations with only horizontal viscosity term $-\Delta_H$;
- For $\gamma < 2$, one obtains the 2D Navier-Stokes equations.

Thus, there are three possible limits of the hydrostatic approximation depending on the assumption on the vertical viscosity. Here, we show how maximal regularity methods and quadratic inequalities - reminiscent of the Fujita-Kato methods - can be an efficient approach to prove norm-convergences in all three cases.

This is a joint work with Ken Furukawa, Yoshikazu Giga, Matthias Hieber, Takahito Kashiwabara, and Marc Wrona, see

https://arxiv.org/abs/2312.03418

for a preprint.