Formation of microstructure for singularly perturbed problems related to helimagnets and shape-memory alloys

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In this talk we present recent results on scalar-valued variational models for pattern formation in helimagnetic compounds and in shape memory alloys. Precisely, we consider a non-convex multi-well bulk energy on the unit square, which favors four gradients $(\pm\alpha, \pm\beta)$, regularized by a singular perturbation in terms of the total variation of the second derivative. We discuss the relation of this continuum model to a discrete $J_1 - J_3$- model via Γ-convergence. Then we derive scaling laws for the minimal energy in the case of an incompatible boundary condition in terms of the singular perturbation parameter as well as the ratio α/β and the incompatibility of the boundary condition. It will be discussed how well-studied models for martensitic microstructure in shape-memory alloys arise as a limiting case and how the presented results can be transferred to the discrete setting. Moreover, we extend the used techniques to understand the domain dependence of the occurrence of microstructure even in the vectorial case.

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