Axisymmetric Vortex Rings at High Reynolds Number

Thierry Gallay (Université Grenoble Alpes)

We consider the unique axisymmetric solution without swirl of the 3D Navier-Stokes equations with singular initial data corresponding to a circular vortex filament. We are interested in the high Reynolds number regime where the kinematic viscosity is small compared to the circulation of the filament. We first construct an approximate solution which can be expanded in powers of a small parameter corresponding to the aspect ratio of the vortex ring. We then show that the exact solution remains close to our approximation over a long time interval, during which the vortex ring moves along its symmetry axis at a speed that was predicted by Kelvin in 1867. To prove this, we introduce self-similar variables located at the (unknown) position of the ring, and we control the evolution of the perturbations using an energy functional related to Arnold's variational characterization of steady states for the 2D Euler equations. This talk is based on joint work with Vladimir Sverak.