Regularity problems for anisotropic models

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The talk is devoted to the study of qualitative properties (potential estimates, asymptotic behavior, boundedness, solution regularity) for elliptic and parabolic anisotropic equations (as well as their variational interpretation) of diffusion-absorption structure with non-standard growth conditions and external sources as well as related variational problems. Main attention will be given to the anisotropic porous medium equations

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(|u|^{m_i - 1} u_{x_i} \right) + f, \tag{0.1}$$

$$1 - \frac{2}{n} < m_1 \le m_2 \le ... \le m_n < \bar{m} + \frac{2}{n},\ \bar{m} = \frac{1}{n} \sum_{i=1}^{n} m_i,$$

$f \in L^1(\Omega_T)$ $\Omega_T = \Omega \times (0,\, T)$, and anisotropic p-Laplace equations

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(|\nabla u|^{p_i - 2} u_{x_i} \right) + f, \tag{0.2}$$

$$\frac{2n}{n+1} < p_1 \le p_2 \le ... \le p_n < 2 + \frac{k}{n},\ \bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i,\ k = n(\bar{p} - 2) + \bar{p}.$$

Such problems serve as mathematical models of many nonlinear processes in anisotropic and inhomogeneous media. The main idea is to construct the analog of the regularity theory for anisotropic elliptic and parabolic equations with external sources via Riesz and Wolff potentials of right-hand side, and to prove local boundedness and continuity of weak solutions.

For some cases it is expected that the regularity theory for nonlinear equations can be linearized via Riesz potentials. Others cases will involve the nonlinear Wolff and parabolic potentials. Interesting observation arises: in all known related publications the cases of slow diffusion (or the degenerate case with $m_i > 1$) and fast diffusion (or the singular case with $m_i < 1$) were considered independently, and methods of proving depended on either fast or slow diffusion takes place. The main novelty here is that we developed the method which allows to avoid former separation into degenerate and singular cases and proved the pointwise estimates for weak solutions of parabolic anisotropic equations via Riesz potential, and as the result, proved the local continuity of weak solutions for all cases.

References

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[2] Buryachenko K., Skrypnik I. Local Continuity and Harnack's Inequality for Double-Phase Parabolic Equations, Potential Analysis, 2022, 56(1), p. 137–164