Mathematical Institute
University Leipzig
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ODE for Physicists - Homework 3

Due: April 26, 2005

7. (10 pts.) Check whether the following ODEs are exact. If so, find the solution curves through the indicated points (x_0, y_0). If possible, give the expressions of these curves in explicit form, $y = y(x)$ or $x = x(y)$.

   (a) $(2 - 9xy^2)x\,\mathrm{d}x + (4y^2 - 6y^3)y\,\mathrm{d}y = 0,$ at $(1, 1)$.

   (b) $\mathrm{e}^{-y}\,\mathrm{d}x - (2y + x\mathrm{e}^{-y})\,\mathrm{d}y = 0,$ at $(5, 0)$.

   (c) $(1 + y^2 \sin 2x)\,\mathrm{d}x - 2y\cos^2 x\,\mathrm{d}y = 0,$ at $(0, 2)$.

8. (6 pts.) (Exercise on partial differentiation.) Fix $d \in \setminus\{1\}$ and consider the function $r\colon \mathbb{R}^d \to \mathbb{R}$ given by

$$r(x) = \|x\|_2 = \sqrt{x_1^2 + x_2^2 + \ldots + x_d^2},$$

   for all $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$. We introduce the *Laplace* operator:

$$\Delta := \sum_{j=1}^{d} \frac{\partial^2}{\partial x_j^2},$$

   i.e., $\Delta f(x) = \sum_{j=1}^{d} \frac{\partial^2}{\partial x_j^2} f(x_1, \ldots, x_d)$ for any $x \in \mathbb{R}^d$ and any function f that is twice differentiable with respect to any x_j.

   (a) For $d = 2$, compute $\Delta(\log r)(x)$ for any $x \in \mathbb{R}^d \setminus \{0\}$ and simplify the expression as far as possible.

   (b) For $d \geq 3$, compute $\Delta(r^{2-d})(x)$ for any $x \in \mathbb{R}^d \setminus \{0\}$ and simplify the expression as far as possible.