FINITE ELEMENT DISCRETIZATIONS OF THE NAVIER-STOKES EQUATIONS

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Abstract

We consider and compare two different discretization methods for solving the stationary, incompressible and non-isothermal Navier-Stokes equations. Modifications of standard Galerkin finite element methods are necessary in order to obtain a stable discretization in case of moderate and high Reynolds and Rayleigh numbers. In particular, we consider the Galerkin/least-squares approach (addition of weighted residuals of the basic equations) and an upwind discretization of the convective terms. The stabilizing effect of both techniques is demonstrated in numerical examples. We conclude with first experiencies with parallel implementation of an upwind method for the isothermal Navier-Stokes equations.

1 CONTINUOUS MODEL

We consider the stationary, incompressible and non-isothermal Navier-Stokes equations in a plane polygonal domain Ω

$$\begin{aligned} -\nu\Delta u + u\cdot\nabla u + \nabla p &= \nu\kappa \mathrm{Ra} T\vec{g} \quad \text{in } \Omega \\ \nabla\cdot u &= 0 \qquad \text{in } \Omega \\ -\kappa\Delta T + u\cdot\nabla T &= 0 \qquad \text{in } \Omega \end{aligned} \tag{1.1}$$

with boundary conditions

$$u = g \quad \text{on} \quad \Gamma, \quad T = \sigma \quad \text{on} \quad \Gamma_D, \quad \frac{\partial T}{\partial n} = 0 \quad \text{on} \quad \Gamma_N. \tag{1.2}$$

Let

$$V = H^1(\Omega)^d, \quad V_0 = H_0^1(\Omega)^d, \quad Q = \{q \in L^2(\Omega) \,:\, (q,1) = 0\},$$
$$S = \{s \in H^1(\Omega) \,:\, s = \sigma \text{ on } \Gamma_D\}, \quad S_0 = \{s \in H^1(\Omega) \,:\, s = 0 \text{ on } \Gamma_D\}$$

and

$$f(T) = \nu\kappa \mathrm{Ra} T\vec{g}.$$

A weak formulation of (1.1), (1.2) reads as follows:
Find $[u,p,T] \in V\times Q\times S$, s. t. $u-g \in V_0$, $T-\sigma \in S_0$ and for all $[v,q,s] \in V_0\times Q\times S_0$

$$\begin{aligned} \nu(\nabla u,\nabla v) + (u\cdot\nabla u, v) - (p,\nabla\cdot v) &= (f(T),v) \\ (q,\nabla\cdot u) &= 0 \\ \kappa(\nabla T,\nabla s) + (u\cdot\nabla T, s) &= 0. \end{aligned} \tag{1.3}$$