Numer. Math. 59, 13–29 (1991)

Numerische Mathematik

A modified streamline diffusion method for solving the stationary Navier–Stokes equation

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Received June 16, 1988

Summary. Recently, Hughes et al. [11, 12] proposed new finite element schemes of Petrov–Galerkin type for solving the Stokes problem which do not require the discrete version of the Ladyshenskaya–Babuška–Brezzi-condition (LBB-condition). In this paper we derive a conforming finite element method for solving the stationary Navier–Stokes equations which combines the advantages of arbitrary finite element spaces for velocity/pressure with the favourable properties of the streamline diffusion method in the case of moderate and high Reynolds number.

Subject classifications. AMS (MOS): 65N30; 76D05; CR: G1.8

1 Introduction

For solving the stationary (Navier-) Stokes equations in the primitive variables many Finite-Element-Codes have been developed up to now. It was recognized early on that the success of mixed methods strongly depends upon the particular pair of velocity and pressure interpolations. So seemingly natural combinations are known to produce oscillating pressures [9]. The critical point in the mathematical theory of mixed methods is the discrete version of the inf-sup or LBB-condition [2, 9]. This is a stability condition which has to be satisfied uniformly with respect to the discretization parameter. It is necessary in the following sense: If it is violated, then the discrete method *must* diverge for at least one (continuous) solution [1].

Recently, Hughes and his co-workers [11, 12] proposed new finite element methods for the Stokes problem which do not require the discrete LBB-condition. The idea consists of combining the usual Galerkin formulation with least-squares forms of the differential equations. It introduces additional stability to the corresponding bilinear form (and thus "circumvents" the inf-sup condition) without perturbing the consistence order of the method (in case of sufficiently smooth solutions). There exists now a growing series of papers on mathematical aspects of this and other regularization techniques, see e.g. [3, 5, 6, 7, 16].

The main objective of this paper is to derive a finite element method for the stationary Navier–Stokes equations which combines the advantages of Hughes–Franca's approach (allowing for arbitrary finite element spaces for velocity and

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pressure) with the favourable pr
case of moderate and high Rey
We refer to streamline diff
Stokes equations using exactly
to the stationary problem usi
velocity and piecewise constant
LBB-condition [15].
For technical reasons (in pa
we use a regularization metho
problem in [3, 4]. It is worth
application of the regularizatio
Tichonov [19]. Hence, in order
effects, one problem is to select
regularization approach to the
to [10].
The plan of the paper is the
important for the following. In
methods for the Stokes proble
a regularized continuous prob
modified streamline diffusion fi
Sect. 6 we study existence and
Finally, the convergence rate in
in Sect. 7.

2 Notations and preliminaries

Throughout this paper, Ω is su
$N = 3$, with a Lipschitz conti
a measurable subset $G \subset \Omega$, l
seminorm on the Sobolev sp
functions $u = (u_1, \ldots, u_N) \in W$
following norms and seminorm

$$\|u\|^p_{k,p,G} = \sum_{l=1}^{N} \|$$

$$\|v\|$$

In case of $G = \Omega$ we omit the
constant independent of h.
In this paper we consider
pressible flow, i.e. we have to
pressure p such that

(2.1) $\quad - \nu\Delta u$