Some achievements in multiscale subgrid modelling

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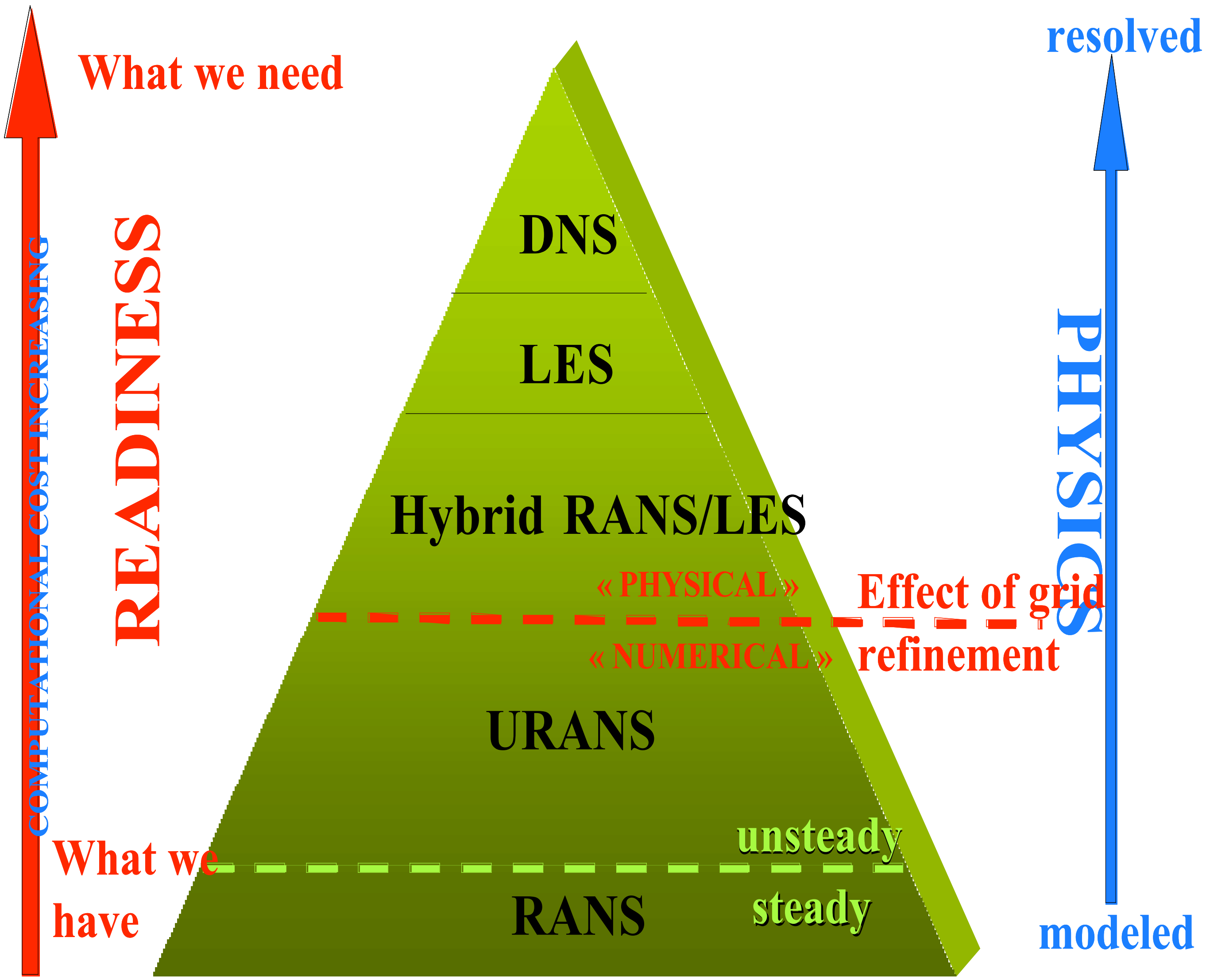
Hierarchy of CFD approaches



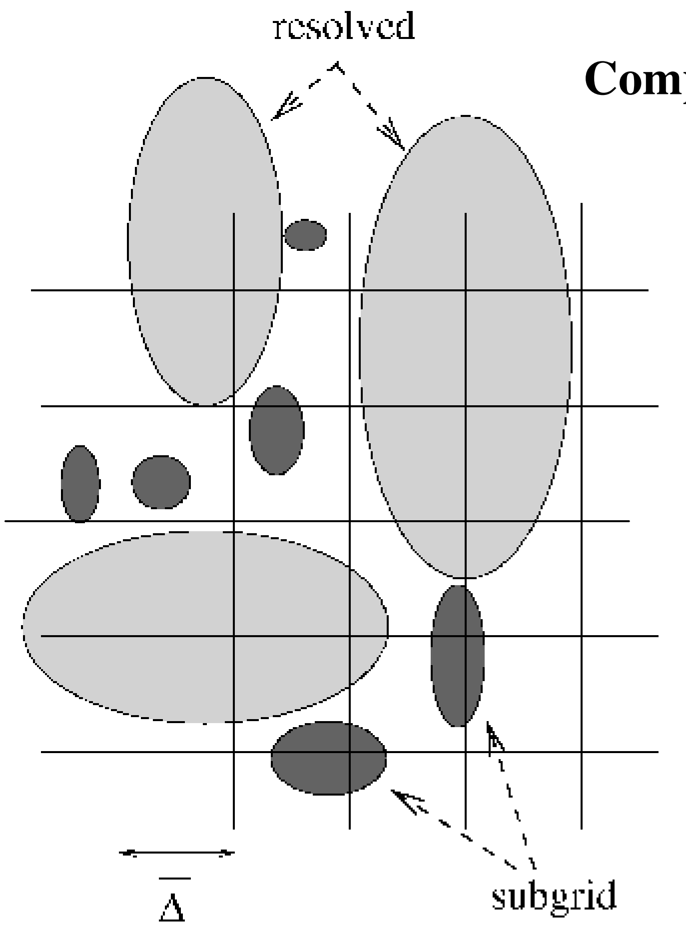
From "**Multiscale and multiresolution approaches in turbulence**"
P.Sagaut, S.Deck, M. Terracol, Imperial College Press, 2006

The LES concept

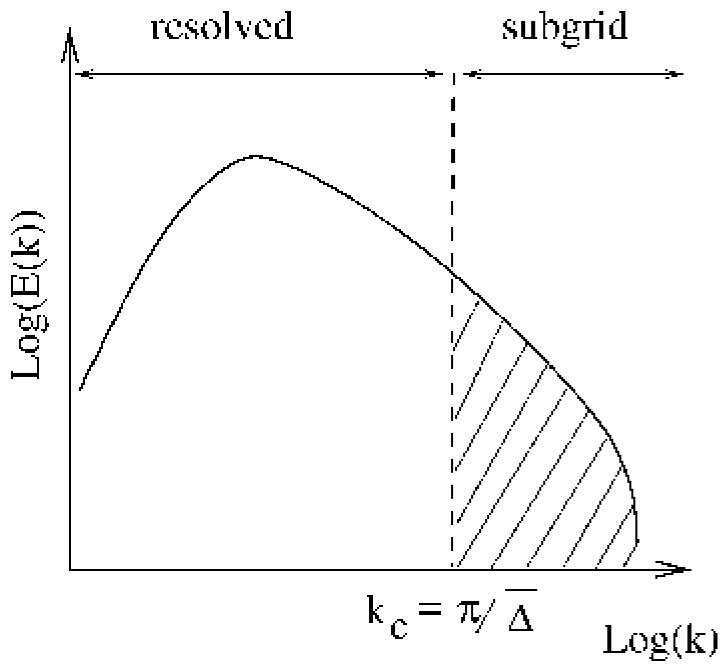
$$\bar{u}(x,t) = G \star u(x,t)$$

Computed 'filtered' solution

Exact solution



PHYSICAL SPACE



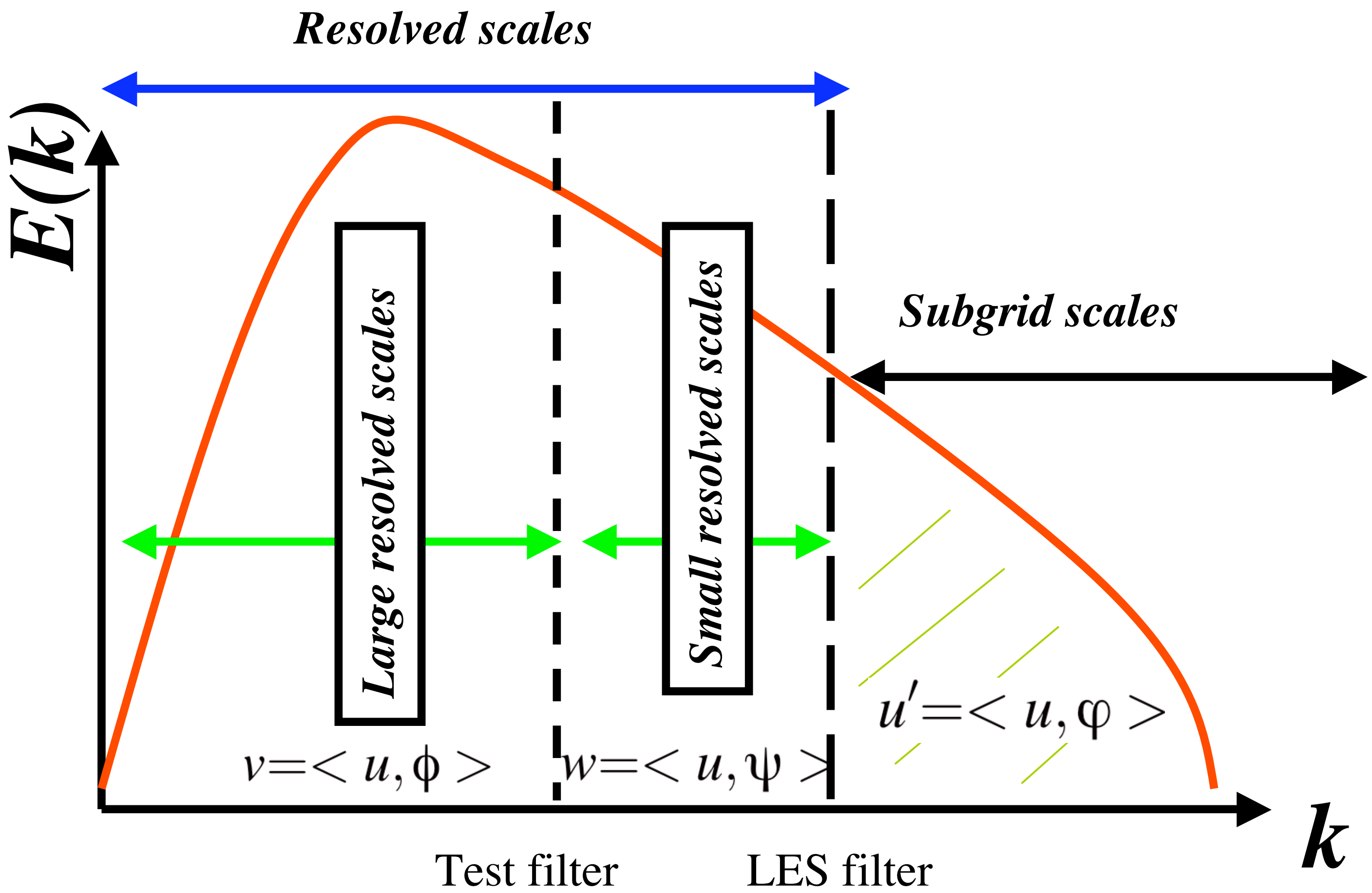
FOURIER SPACE

Overview

- Brief reminder about VMS-LES methods
- A few examples of use at UPMC
- A remark dealing with numerical errors/subgrid model coupling
- On the value of the constant in VMS Smagorinsky model

Schematic view of VMS-LES strategies

$$\frac{\partial u}{\partial t} + F(u,u) = 0 \qquad u = \bar{u} \oplus u' = v \oplus w \oplus u'$$



Schematic view of VMS-LES strategies

Neglected (or modeled)

$$\frac{\partial v}{\partial t} + < F(\bar{u},\bar{u}),\phi > = -2 < F(\bar{u},u'),\phi > - < F(u',u'),\phi >$$

$$\frac{\partial w}{\partial t} + < F(\bar{u},\bar{u}),\psi > = -2 < F(\bar{u},u'),\psi > - < F(u',u'),\psi >$$

modeled

$$\frac{\partial u'}{\partial t} + \langle F(u,u),\varphi \rangle = 0$$

Ignored (or simplified PDEs ?)

General form of VMS subgrid viscosities

Subgrid viscosity — Strain tensor — Selected subspace

$$r.h.s. = \langle \nu_t(h,X)\,(\nabla Y + \nabla^t Y), Z \rangle$$

Viscosity test field:

- $X=v$: Large - ??? Model
- $X=w$: Small - ??? Model
- $X=v+w$: All - ??? Model

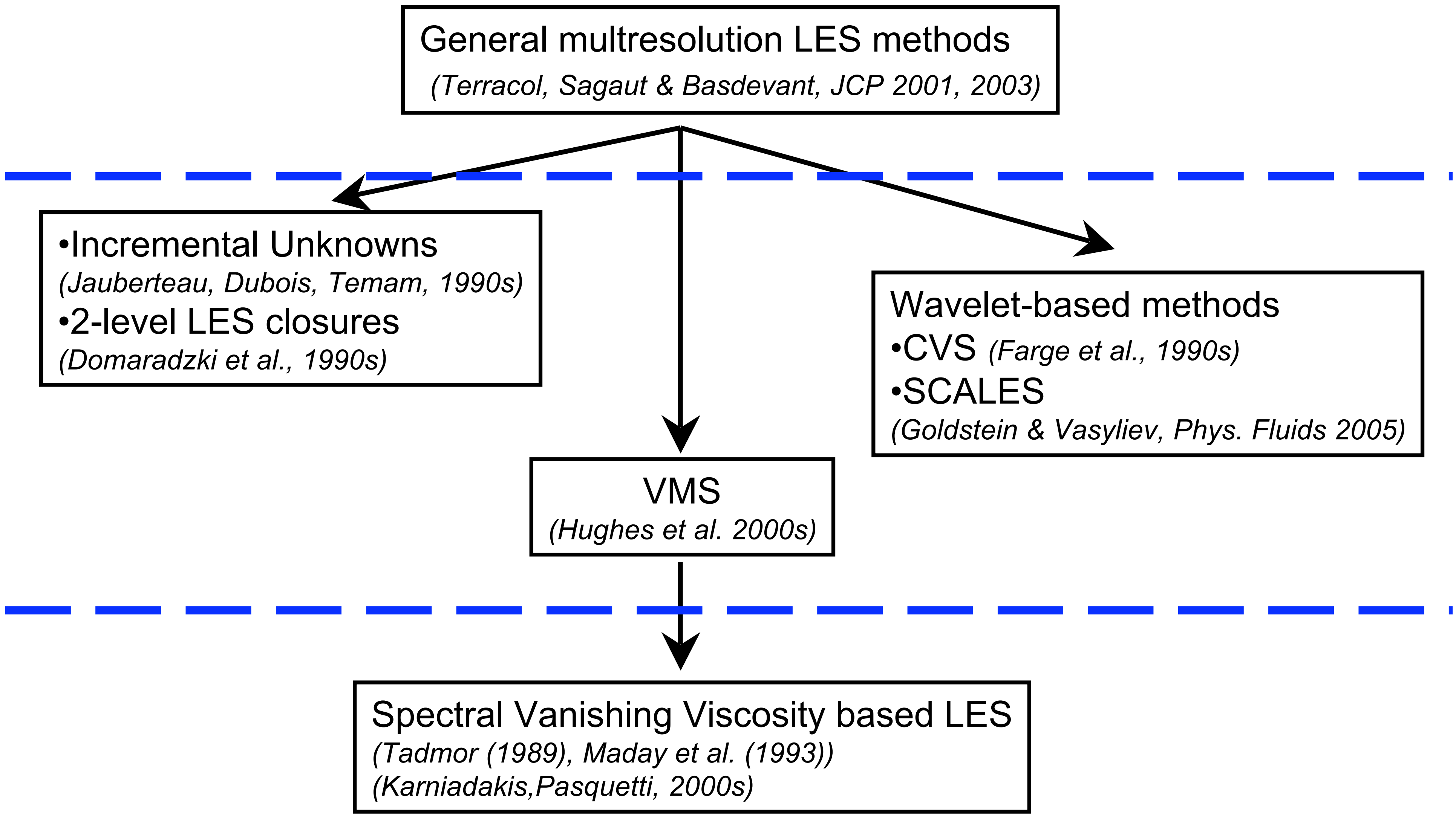
Strain test field:

- $Y=v$: ???- Large Model
- $Y=w$: ???-Small Model
- $Y= v+w$: ???- All Model

Target subspace:

- $Z=\phi$
- $Z= \psi$
- $Z= \phi + \psi$

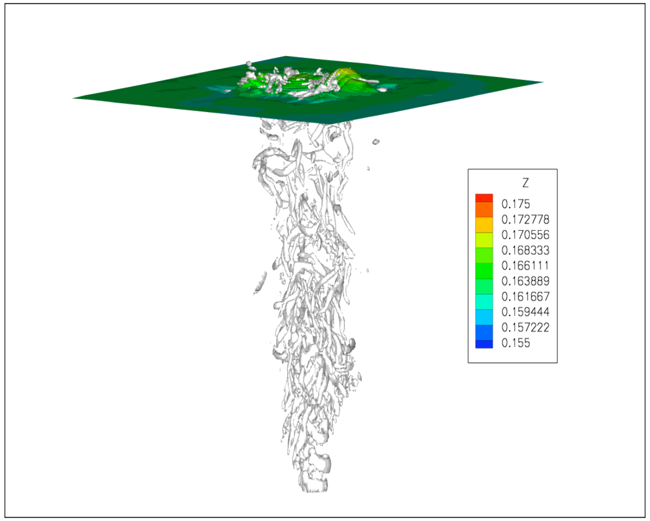
Hierarchy of multiscale/level LES methods



Examples of use at UPMC

Jet/free surface interaction

(with TREFLE Lab.)



Numerical methods	Physical model	Flow	Reference
Spectral method	incompressible	Isotropic turbulence	*Sagaut & Levasseur, Phys. Fluids, 2005*
Spectral/FD	incompressible	Channel flow	*Meyers & Sagaut, Phys. Fluids,2007*
Unstructured GLS	compressible	Isotropic turbulence	*Levasseur & al., CMAME*
Unstructured GLS	compressible	Weapon bay	*Levasseur & al., JFS, in press*
Unstructured FV	compressible	Isotropic turbulence, channel flow	*Sagaut & Ciardi, Phys. Fluids, 2006*
Stpectral method	incompressible	Free surface channel	*Reboux & al. Phys. Fluids, 2006*
Structured FD	incompressible	Free surface flow	*Moreau & al., submitted*

Numerical error/subgrid model interactions

$$\frac{\delta u_h}{\delta t} + F_h(u_h, u_h) = 0$$

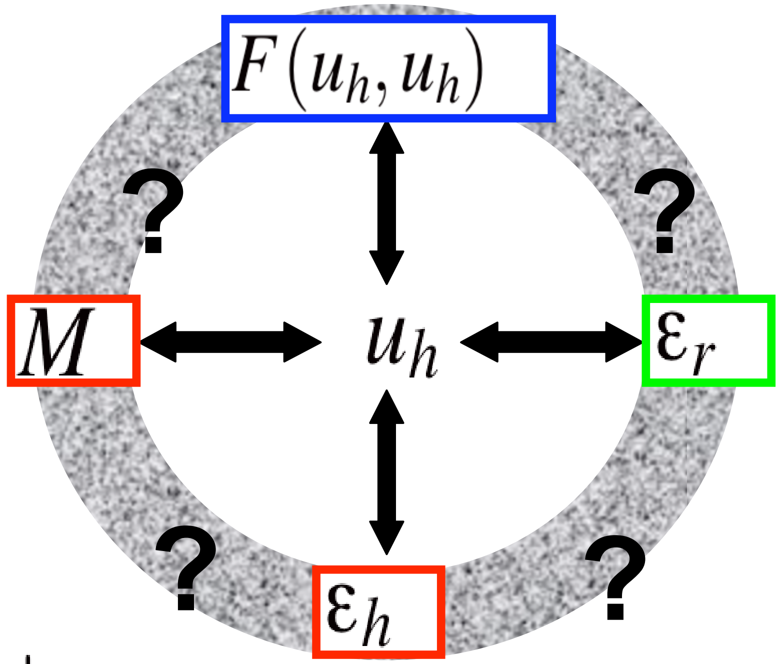
$$\Downarrow$$

$$\frac{\partial u_h}{\partial t} + F(u_h, u_h) = M + \varepsilon_r + \varepsilon_h$$

Exact subgrid model

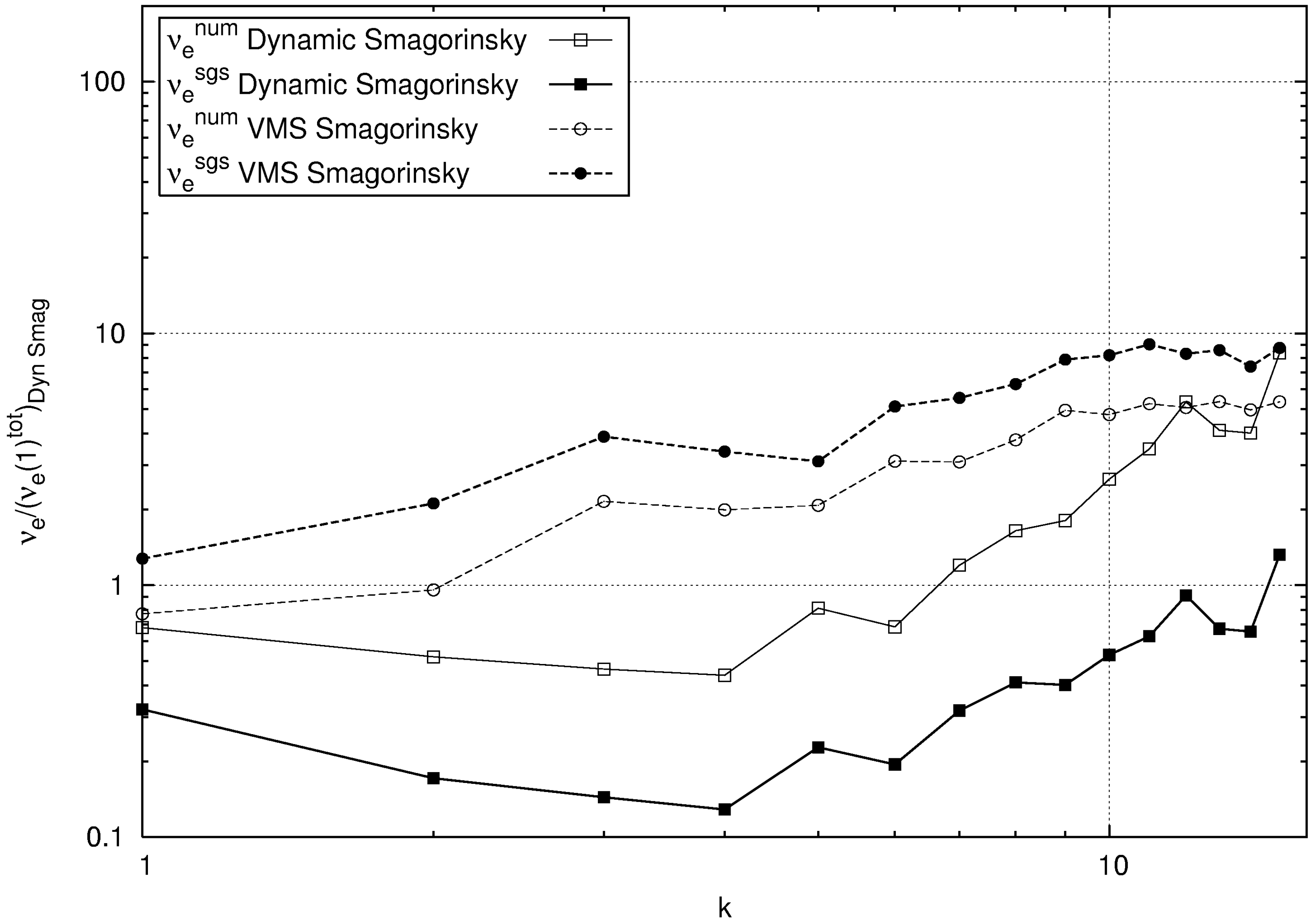
Subgrid modelling error

numerical error



Numerical error/subgrid model interactions

(Ciardi & al., J. Comput. Phys., 2005)
(Sagaut & Ciardi, Phys. Fluids, 2006)



Subgrid viscosity model consistency

(Meyers & Sagaut, J. Fluid Mech., 2006)

- A consistent subgrid model must account for exact energy transfers between resolved and subgrid scales

  ➪ sensitivity to two non-dimensional parameters

$$L/\Delta \qquad \eta/\Delta$$

- Classical subgrid models designed for (asymptotic canonical case)

$$L/\Delta \gg 1 \quad \eta/\Delta \ll 1$$

➪ problems in realistic cases, DNS not recovered satisfactorily

Accounting for realistic spectrum shape

Pope's spectrum model *(Pope, 2000)*

$$E(k) = K_0 \varepsilon^{2/3} k^{-5/3} f_L(kL) f_\eta(kLRe_L^{-3/4})$$

$$f_L(x) = \left(\frac{x}{\sqrt{x^2 + c_L}} \right)^{11/3}$$

Large scale part (flow-dependent)

$$f_\eta(x) = \exp\left(-c_\beta ((x^4 + c_\eta^4)^{1/4} - c_\eta) \right)$$

Viscous range

Inertial Range consistent VMS models

Small-Small VMS Smagorinsky

- Subgrid model

$$m_{ij} = -\left[2C_{s1}^2 \Delta^2 |\overline{S}'| \overline{S}'_{ij}\right]'$$

- Induced subgrid dissipation

$$\varepsilon_{t,s1} = (C_{s1}\Delta)^2 \langle 2\overline{S}'_{ij}\overline{S}'_{ij}\rangle^{3/2} = (C_{s1}\Delta)^2 \left(2\int_0^\infty k^2 (H'(k))^2 \overline{E}(k)\,\mathrm{d}k\right)^{3/2}$$

Test filter

Resolved field spectrum

Viscous effects and total dissipation

Normalized total dissipation

$$1 = \frac{\varepsilon_t}{\varepsilon} + \frac{\varepsilon_\nu}{\varepsilon}$$

$$1 = \gamma_1^2 C_{s1}^2 \left(\frac{3\alpha}{2}\right)^{3/2} \pi^2 \left(1 - \beta^{4/3}\right)^{3/2} \Psi_1^{3/2} + \left(\frac{3\alpha}{2}\right) \frac{(\gamma \pi L/\Delta)^{4/3}}{Re_L} \Phi$$

Viscous dissipation
neglected in usual analysis

Inertial-range consistency requires to account for this contribution

Scaling functions

$$\beta = \Delta'/\Delta$$

$$\gamma = \frac{\Delta}{\pi}\left(\frac{4}{3}\int_0^\infty x^{1/3} G^2(x) dx\right)^{3/4} \qquad \gamma_1 = \left(\frac{4}{3}\frac{\int_0^\infty k^{1/3}(H'(k))^2 (G(k))^2 \,\mathrm{d}k}{(\pi/\Delta)^{4/3}\left(1-\beta^{4/3}\right)}\right)^{3/4}$$

$$\Phi(L/\Delta, Re_L) = \frac{4}{3}\frac{1}{(\gamma\pi L/\Delta)^{4/3}}\int_0^\infty x^{1/3} G^2(x/L) f_L(x) f_\eta(x Re_L^{-3/4}) dx$$

$$\Psi_1\left(\frac{L}{\Delta}, Re_L\right) = \frac{4}{3}\frac{\int_0^\infty x^{1/3}(H'(x/L))^2 (G(x/L))^2 f_L(x) f_\eta(x Re_L^{-3/4})\,\mathrm{d}x}{\int_0^\infty x^{1/3}(H'(x/L))^2 (G(x/L))^2 \,\mathrm{d}x}$$

➪ filter & spectrum dependent

IRC Small-Small model

$$C_{s1} = \frac{C_{s,\infty}}{\gamma_1} \frac{\Psi_1^{-3/4}}{\left(1-\beta^{4/3}\right)^{3/4}} \sqrt{1-\left(\frac{\gamma L}{C_{s,\infty}\Delta}\right)^{4/3} \frac{1}{Re_L}\Phi}$$

$$= \frac{C_{s,\infty}}{\gamma_1} \frac{\Psi_1^{-3/4}}{\left(1-\beta^{4/3}\right)^{3/4}} \sqrt{1-\left(\frac{\gamma \eta}{C_{s,\infty}\Delta}\right)^{4/3} \Phi}.$$

➪ **a single universal value doesn't exist !**

All-Small VMS Smagorinsky

- Subgrid model

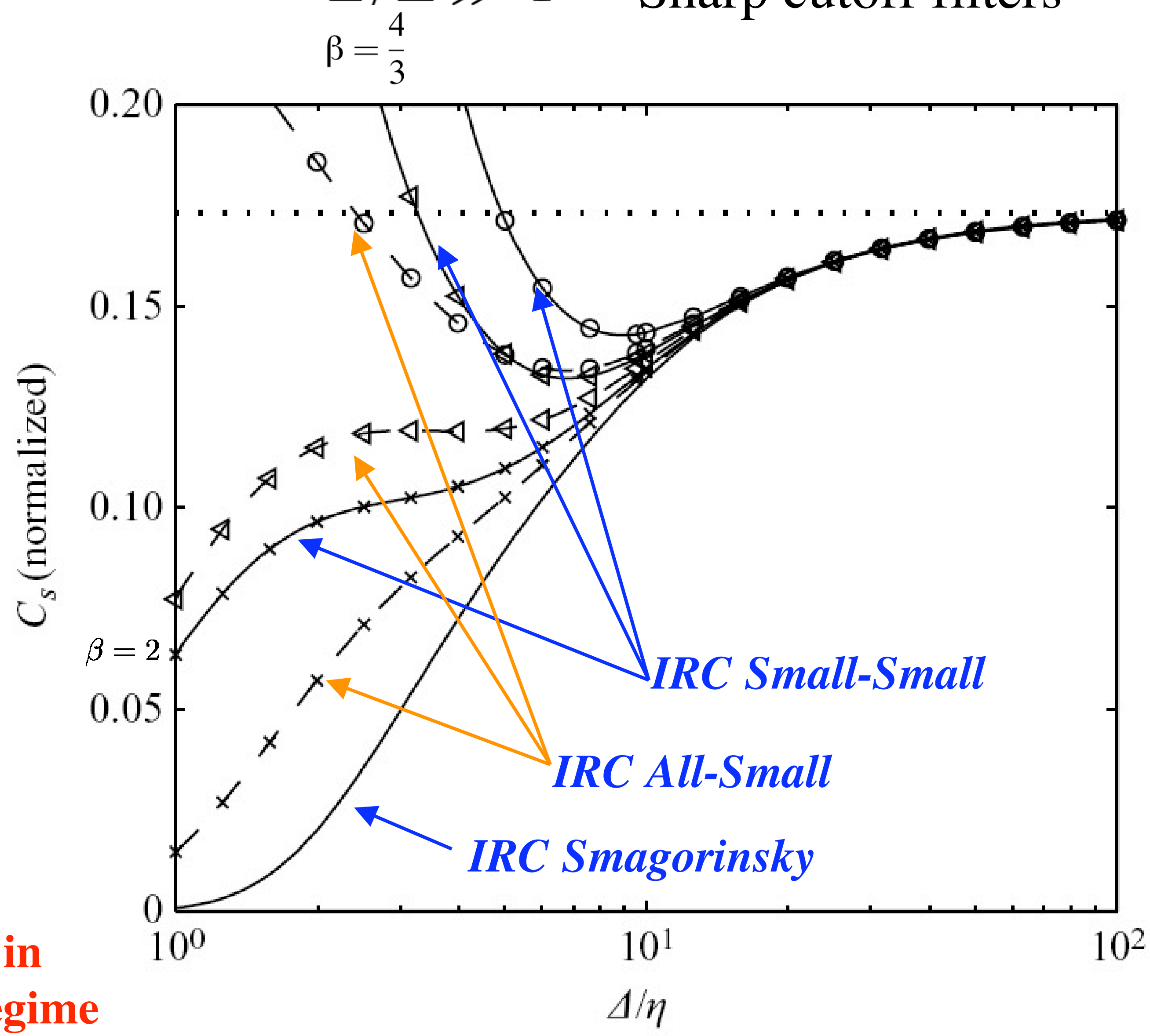
$$m_{ij} = -\left[2C_{a1}^2 \Delta^2 |\overline{S}| \overline{S}'_{ij}\right]'$$

- IRC variant

$$\begin{aligned} C_{a1} &= C_{s,\infty} \frac{\Phi^{-1/4} \Psi_1^{-1/2}}{\gamma^{1/3} \gamma_1^{2/3} \sqrt{1-\beta^{4/3}}} \sqrt{1 - \left(\frac{\gamma L}{C_{s,\infty} \Delta}\right)^{4/3} \frac{1}{Re_L} \Phi} \\ &= C_{s,\infty} \frac{\Phi^{-1/4} \Psi_1^{-1/2}}{\gamma^{1/3} \gamma_1^{2/3} \sqrt{1-\beta^{4/3}}} \sqrt{1 - \left(\frac{\gamma \eta}{C_{s,\infty} \Delta}\right)^{4/3} \Phi}. \end{aligned}$$

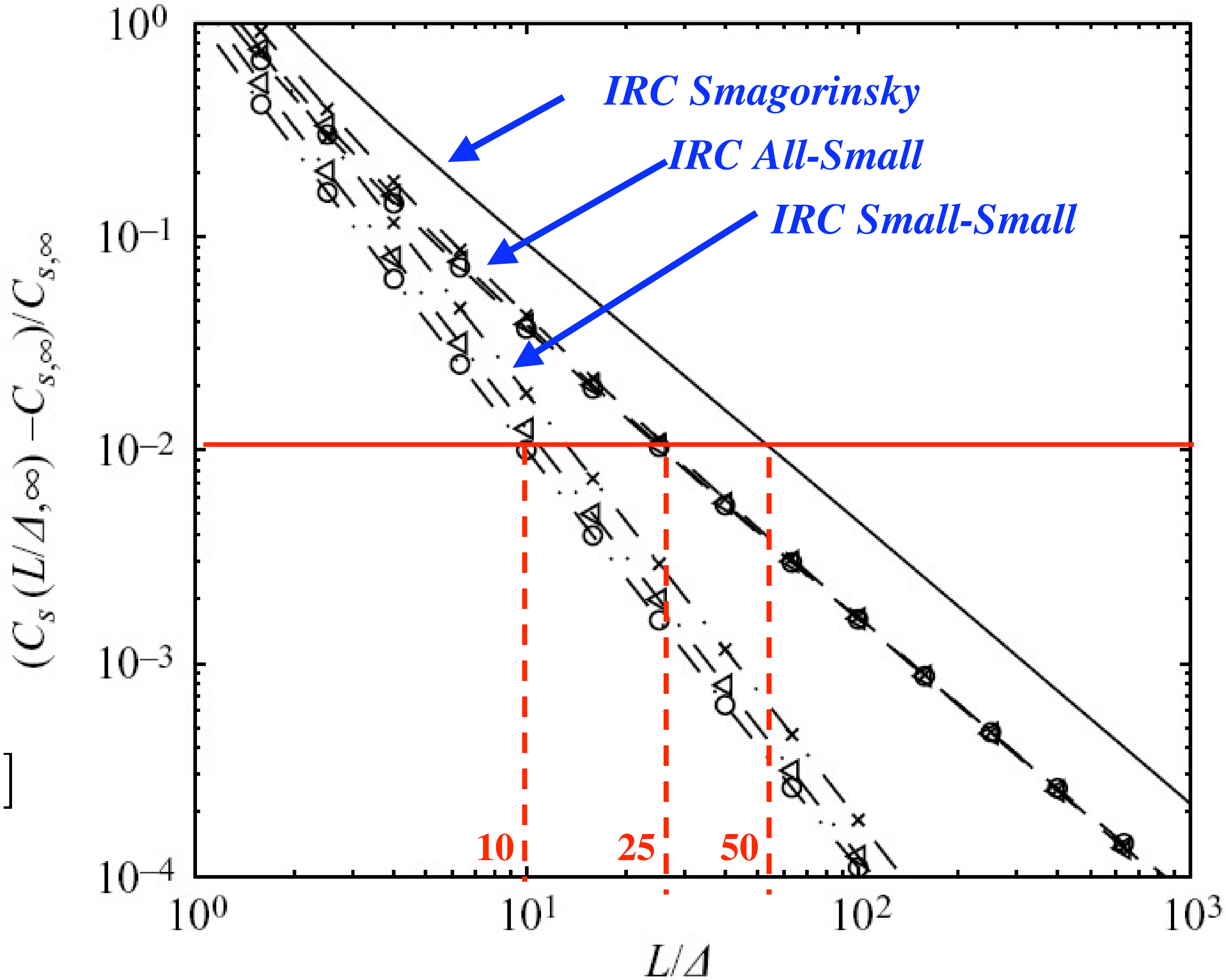
IRC constant behavior

$L/\Delta \gg 1$ Sharp cutoff filters



Transition in Quasi-DNS regime

Deviation from asymptotic value



$\beta \in [1, 2]$

Asymptotic behavior

$$L/\Delta \gg 1 \quad Re_L = +\infty$$

$$\delta\Phi = \frac{4}{3}\frac{\int_0^{\pi L/\Delta} x^{1/3}(1 - f_L(x))\,\mathrm{d}x}{(\pi L/\Delta)^{4/3}}$$

$$\delta\Phi \approx \frac{4}{3}\frac{\int_0^{\pi L/\ell} x^{1/3}(1 - f_L(x))\,\mathrm{d}x + \int_{\pi L/\ell}^{\pi L/\Delta} C x^{1/3} x^{-p}\,\mathrm{d}x}{(\pi L/\Delta)^{4/3}}$$

$$= C'\,(L/\Delta)^{-4/3} + C''\,(L/\Delta)^{-p}\,,$$

$$\delta C_s \sim (L/\Delta)^{-\min(4/3,\,p)}$$
$$\delta C_{a1} \sim (L/\Delta)^{-\min(4/3,\,p)}$$
$$\delta C_{s1} \sim (L/\Delta)^{-p}.$$

$p \in [1,2]$

Practical implementation: remapping

- IRC models not tractable ⇨ approximations are necessary

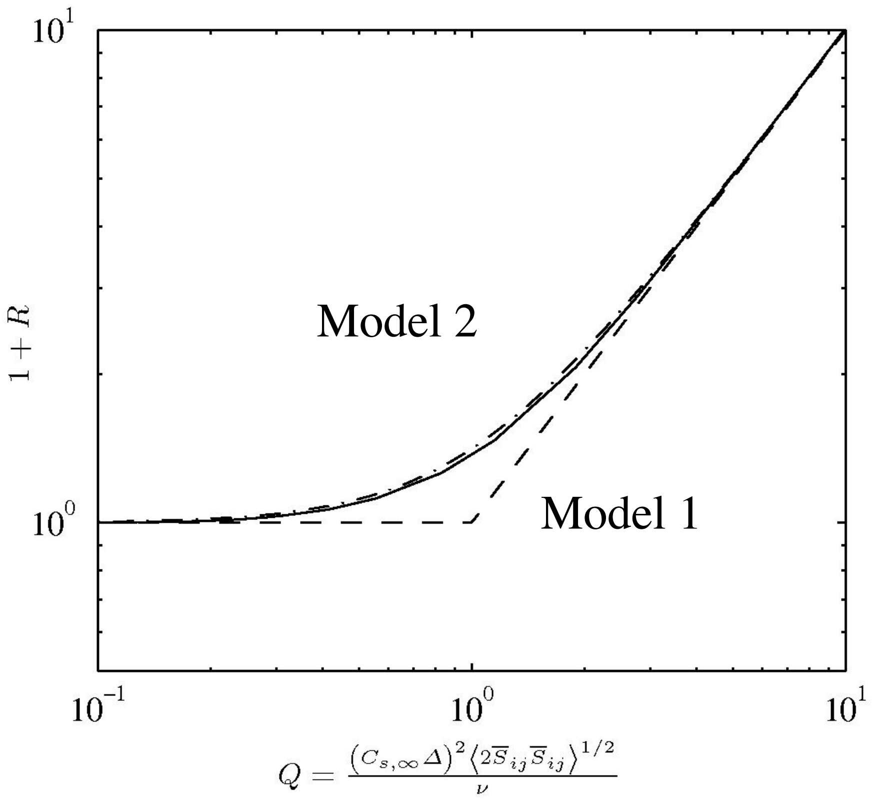
- Introducing 2 non-dimensional parameter (e.g. Smagorinsky model)

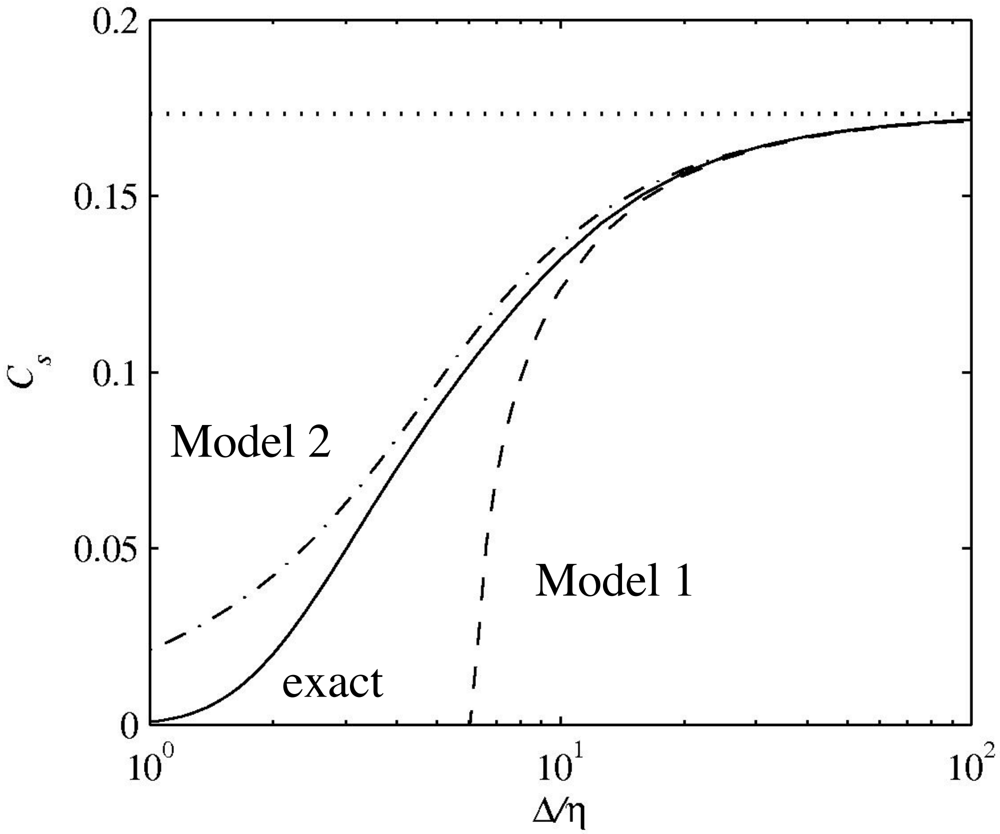
$$R = \frac{\nu_t}{\nu} \qquad Q = \frac{(C_\infty \Delta/\gamma)^2 \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}}{\nu}$$

A closure is expressed as *F(R,Q)=0*

Original model	$R = Q \Longleftrightarrow \nu_t = \nu_{Lilly}$
Model 1:	$R = \max(Q-1,0) \Longleftrightarrow \nu_t = \max(\nu_{Lilly} - \nu, 0)$
Model 2:	$(R+1)^2 - Q^2 - 1 = 0 \Longleftrightarrow \nu_t = \sqrt{\nu_{Lilly}^2 + \nu^2} - \nu$

Remapping for Smagorinsky model





Quadratic remapping for VMS models

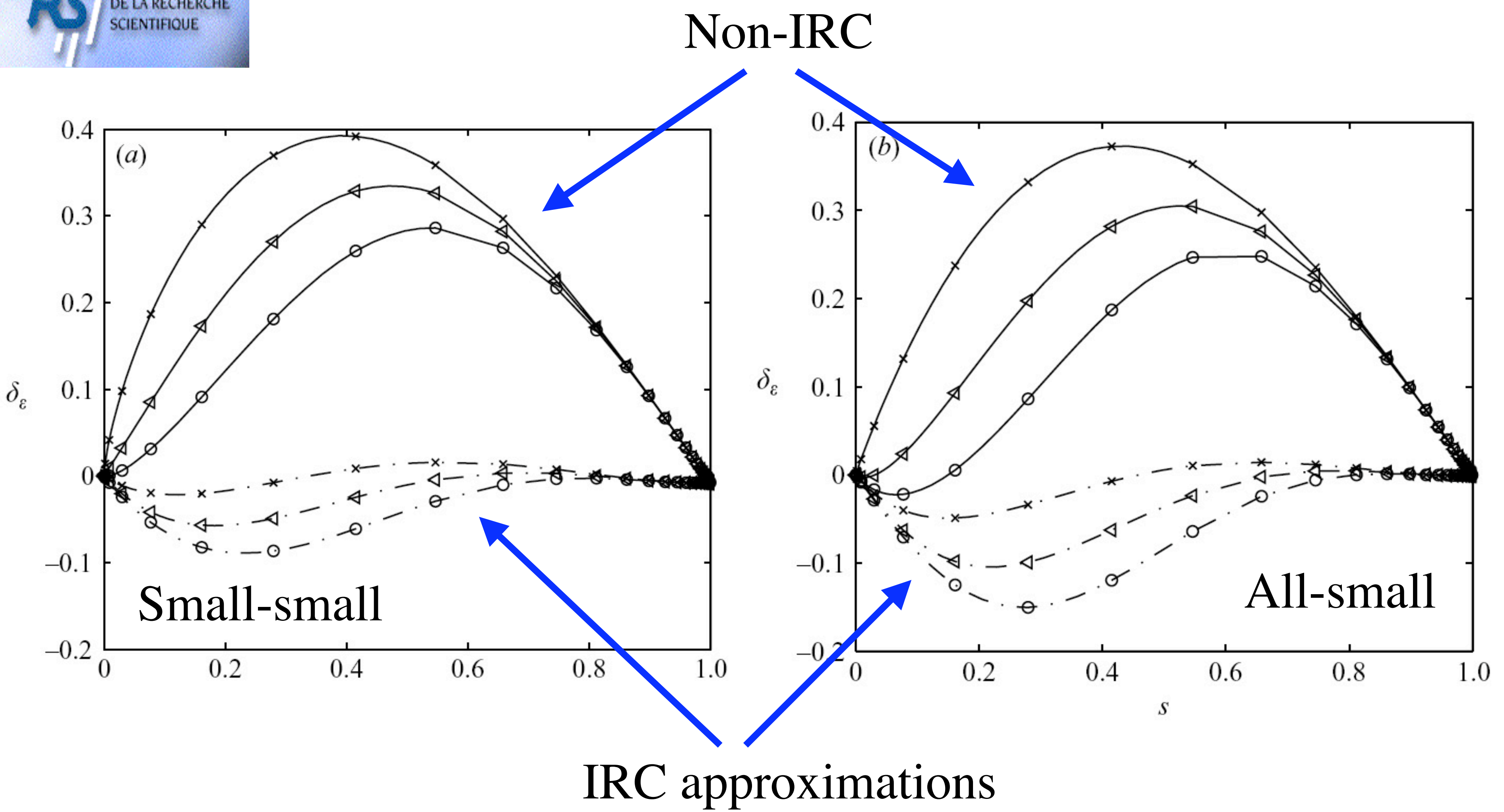
- Small-Small model (quadratic mapping)

$$\nu'^{*}_{t,s1} = \frac{(\gamma/\gamma_1)^{4/3}}{1-\beta^{4/3}} \left(\sqrt{\left(\frac{C_{s,\infty}\Delta}{\gamma}\right)^4 \frac{(\gamma/\gamma_1)^{4/3}\langle 2\overline{S}'_{ij}\overline{S}'_{ij}\rangle}{1-\beta^{4/3}} + \nu^2} - \nu \right)$$

- All-Small model (quadratic mapping)

$$\nu'^{*}_{t,a1} = \frac{(\gamma/\gamma_1)^{4/3}}{1-\beta^{4/3}} \left(\sqrt{(C_{s,\infty}\Delta/\gamma)^4 \langle 2\overline{S}_{ij}\overline{S}_{ij}\rangle + \nu^2} - \nu \right)$$

Normalized dissipation error



Conclusions

- Inertial-range consistent VMS subgrid models defined
- Asymptotic behavior may be complex !
- Recovery of DNS is model-dependent
- IRC approach can be applied to any VMS models
- IRC analysis may be extended considering numerical dissipation

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Thank you!