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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 09

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof. If tools from AI are used to solve the problems, then this has to be indicated.

1. *Norm for a non-conforming finite element space.* Let P_1^{nc} be the two-dimensional Crouzeix–Raviart finite element space with functions that vanish in the midpoints of the edges which lie on the boundary of the domain. Prove that

$$\|v_h\|_h = \left(\sum_{K \in \mathcal{T}_h} \|\nabla v_h\|_{L^2(K)}^2 \right)^{1/2}$$

   defines a norm in this space.
   Hint: It is clear that $\|v_h\|_h$ defines a seminorm. One has to show that from $\|v_h\|_h = 0$, it follows that $v_h = 0$. **2 points**

2. Show the estimate given in Lemma 6.4, with the conditions on the domain given in this lemma, for $p = \infty$, $k = 0$, and $l = 1$. Start with estimate (6.2), i.e., with

$$|v(\boldsymbol{x})| \le \frac{2R}{|\Omega|} \int_\Omega \int_0^1 \|\nabla v(t\boldsymbol{x} + (1-t)\boldsymbol{y})\|_2 \ dt\ d\boldsymbol{y}.$$

   2 points

3. Do not forget the programming problem from exercise sheet 08.

The exercise problems should be solved in groups of four students. The solutions have to be submitted until **Monday, June 30, 2025, 10:00 a.m.** via the whiteboard.