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## Numerical Mathematics III – Partial Differential Equations Exercise Problems 04

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof. If tools from AI are used to solve the problems, then this has to be indicated.

1. Matrix property, detail of the proof of Theorem 2.45. In the proof of Theorem 2.45, the following argument is used: Let  $A, B \in \mathbb{R}^{n \times n}$  be two symmetric and positive definite matrices with AB = BA and

$$(A\underline{x},\underline{x}) \ge (B\underline{x},\underline{x}) \quad \forall \ \underline{x} \in \mathbb{R}^n,$$

then  $||A\underline{x}||_2 \ge ||B\underline{x}||_2$  for all  $\underline{x} \in \mathbb{R}^n$ . The symbols denote the Euclidean inner product and the Euclidean vector norm. Prove this statement. **2 points** 

2. Code for nine point stencil. The problem is formulated analogously as Problem 4 from exercise sheet 02. The only difference is that the finite difference scheme with the nine point stencil should be used (instead of using the five point stencil). 4 points

The exercise problems should be solved in groups of four students. The solutions have to be submitted until Monday, May 19, 2025, 10:00 a.m. via the white-board.