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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 04

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Matrix property, detail of the proof of Theorem 2.45.* In the proof of Theorem 2.45, the following argument is used: Let $A, B \in \mathbb{R}^{n \times n}$ be two symmetric and positive definite matrices with $AB = BA$ and

$$(A\underline{x}, \underline{x}) \geq (B\underline{x}, \underline{x}) \quad \forall \underline{x} \in \mathbb{R}^n,$$

then $\|A\underline{x}\|_2 \geq \|B\underline{x}\|_2$ for all $\underline{x} \in \mathbb{R}^n$. The symbols denote the Euclidean inner product and the Euclidean vector norm. Prove this statement. **2 points**

2. *Code for nine point stencil.* The problem is formulated analogously as Problem 4 from exercise sheet 02. The only difference is that the finite difference scheme with the nine point stencil should be used (instead of using the five point stencil). **4 points**

The exercise problems should be solved in groups of four to five students. The solutions have to be submitted until **Monday, May 22th, 2023, 10:00 a.m.** via the whiteboard.