Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 11 (final exercise sheet)

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Modeling using a ordinary differential equation.* A motor boat moves over water at rest with constant velocity of 20 km/h. At this speed the motor of the boat stops and the velocities continuously drops to 7 km/h within a period of 30 seconds. The water is assumed to decelerate the boat proportional the velocity of the latter. Compute the velocity of the boat 3 minutes after the motor stops. How far does the boat move after 2 minutes after the motor stops. **4 points**

2. *Integrable classes of ordinary differential equations of first order.*

   (a) Determine the general solution for the following differential equation

   $$xy'(x) - 4y(x) - x^2\sqrt{y(x)} = 0.$$

   (b) Solve the following initial value problem

   $$y'(x) = \frac{-x+2}{x(1-x)}y(x) + \frac{1}{x^2(x-1)}y^2(x), \quad y(2) = a,\ a \in \mathbb{R},\ a > 0.$$

   5 points

3. *Integrable classes of ordinary differential equations of first order.* Determine the general solution to the following differential equation.

   $$4y'(x) + y^2(x) + 4x^{-2} = 0.$$

   This equation has a solution of the form $z_0(x) = \frac{a}{x}$. **4 points**

4. *Iteration of Picard-Lindelöf.* Approximate the Solution of the initial value problem

   $$\begin{aligned}(1+x)y'(x) + y(x) &= (1+x)^{-1},\\ y(0) &= 1,\end{aligned}$$

   using the Picard iteration. Starting from the initial value compute the forth iterate. For the last iterate compute the absolute value of the approximation error with respect to the analytical solution

   $$y(x) = \frac{\ln(x+1)+1}{1+x}$$

   for $x = 0.5$. **3 points**

The exercises should be solved in groups of two students. They have to be submitted until Sie **Monday, 08.07.2024, 10:00**, either in the box of the tutor or electronically via whiteboard.