Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 10

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Vector field.* Sketch the vector field for the ordinary differential equation $y'(x) = x^2 + y^2(x)$.

   2 points

2. *Types of integrable first order ordinary differential equations.* Solve the following ordinary differential equations.

   a) $y'(x) + y^2(x) = 1,$
   b) $y'(x) + y(x)\cos x = 0,$
   c) $y'(x) = xy^7(x),$
   d) $2y(x)y'(x) = x^2.$

   4 points

3. *Solution by substitution.* Determine the general solution of

   $$y'(x) = (x - y(x))^2 + 1.$$

   Hint: Find a suitable substitution. **3 points**

4. *Initial value problem with multiple solutions.* Demonstrate that solutions of the initial value problem

   $$y'(x) = \sqrt{|y(x)|}, \quad y(0) = 0$$

   are not unique. **4 points**

The exercises should be solved in groups of two students. They have to be submitted until Sie **Monday, 01.07.2024, 10:00**, either in the box of the tutor or electronically via whiteboard.