Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Berlin, 13.05.2024

Numerik I

English translation of Übungsserie 05

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Hessenberg matrix.* Find a matrix $A \in \mathbb{R}^{n\times n}$, which is non zero in its upper triangle and the first secondary diagonal only. This matrix is called (upper) Hessenberg matrix and can be transformed efficiently into upper triangular shape by a Givens rotation.

   Transform the following Hessenberg matrix
   $$H = \begin{pmatrix} 3 & 5 & 1 & 7 \\ 4 & 2 & 0 & -4 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 2 & -2 \end{pmatrix}$$
   into upper triangular shape by Givens rotations (correct to four decimal places). **3 Punkte**

2. *Repetition of polynomial interpolation.*

   (a) Prove
   $$\frac{\omega_{n+1}(x)}{(x-x_i)\omega'_{n+1}(x_i)} = \prod_{j=0, j\neq i}^{n} \frac{x-x_j}{x_i-x_j},$$
   where $\omega_{n+1}(x)$ is the nodal polynomial
   $$\omega_{n+1}(x) = \prod_{j=0}^{n}(x-x_j).$$

   (b) Let $(-2,3)$, $(-1,10)$ and $(1,5)$ be points in $\mathbb{R}^2$. Compute the second degree interpolation polynomial connecting these points.

   2+2 points

3. *Hermite interpolation.*

   (a) Determine the Hermite interpolation polynomial $p \in P_3$, which satisfies
   $$p(-1) = -13, \quad p'(-1) = 14, \quad p''(-1) = -22, \quad p'''(-1) = 18.$$

   (b) Let $n \in \mathbb{N}$, $n \geq 0$, and $c_i \in \mathbb{R}$ for $i \in \{0,\dots,n\}$. Prove the following statement. The Hermite interpolation polynomial on $[-1,1]$ which satisfies
   $$p^{(i)}(-1) = c_i, \quad p^{(i)}(1) = (-1)^i c_i, \quad i = 0,\dots,n,$$
   is element of $U = \operatorname{span}\left\{1, x^2, \dots, x^{2n}\right\}$ and uniquely determined.

2+4 points

The exercises should be solved in groups of two students. They have to be submitted until Sie **Monday, 27.05.2024, 10:00**, either in the box of the tutor or electronically via whiteboard.