Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 04

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Properties of the Householder transformation.* Prove the following properties of the Householder transformation.
   Let $\boldsymbol{u} \in \mathbb{R}^m$ such that $\|\boldsymbol{u}\|_2 = 1$ and $H = I - 2\boldsymbol{u}\boldsymbol{u}^T \in \mathbb{R}^{m\times m}$. Then
   i) $H = H^T$,
   ii) $H^2 = I$,
   iii) $H^T H = I$,
   iv) $H\boldsymbol{y} = \boldsymbol{y}$, $\boldsymbol{y} \in \mathbb{R}^m$, is equivalent to $\boldsymbol{y}^T\boldsymbol{u} = 0$,
   v) and $H\boldsymbol{u} = -\boldsymbol{u}$.

   3 points

2. Let $\mathbf{x} = (9, 2, 6)^T$ and $\boldsymbol{y} = (-11, 0, 0)^T$.
   (a) Find the Householder transformation H such that $H\mathbf{x} = \boldsymbol{y}$.
   (b) Find nontrivial (i.e. $\neq 0$) vectors $\boldsymbol{w}$ and $\boldsymbol{z}$ such that
   $$H\boldsymbol{w} = -\boldsymbol{w} \quad \text{and} \quad H\boldsymbol{z} = \boldsymbol{z}.$$

   3 points

3. *Programming exercise for the least squares method* To determine the functional dependence of work ours per capita and the number of the number of surgical interventions, appropriate data of 15 hospitals was collected.

y (monthly work hours)	x (number of surgical interventions)
1275	230
1350	235
1650	250
2000	277
3750	522
4222	545
5018	625
6125	713
6200	735
8150	820
9975	992
12220	1322
12750	1900
13014	2022
13275	2155

For the functional dependence of work hours and the number of surgical interventions there are three models available.

(a) $y = a + bx$,

(b) $\ln(y) = a + b/x$,

(c) $y = a + bx + cx^2$.

- Write a program to solve the three implied linear curve fitting problems. For the solution use the QR decomposition (in Matlab `qr`, in python3 `numpy.linalg.qr`).
- Determine the unknown coefficients in the models.
- Compute the error of least squares for each model. Which one is the best?
- Depict your results graphically.

6 points

The exercises should be solved in groups of two students. They have to be submitted until Sie **Monday, 13.05.2024, 10:00**, either in the box of the tutor or electronically via whiteboard.