Department for Mathematics and Computer Science
Freie Universität Berlin
Prof. Dr. V. John, john@wias-berlin.de
André-Alexander Zepernick, a.zepernick@fu-berlin.de

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Numerical Mathematics II

Exercise Problems 10

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Stability function of* `ode23s`. Consider the linearly implicit Runge–Kutta method `ode23s`

$$
\begin{aligned}
(I - ahJ)\,K_1 &= f(y_k), \quad a = \frac{1}{2+\sqrt{2}},\\
(I - ahJ)\,K_2 &= f\left(y_k + \frac{1}{2}hK_1\right) - ahJK_1,\\
y_{k+1} &= y_k + hK_2
\end{aligned}
$$

   with $J = f_y(y_k) = f'(y_k)$. Show that the stability function of this method, for sufficiently small step sizes h, is

$$
R(z) = \frac{1 + (1-2a)z}{(1-az)^2}.
$$

   Hint: It suffices to consider an autonomous equation. **3 points**

2. *3-step Adams–Bashforth method.* Derive the 3-step Adams–Bashforth method ($q = 3$). **2 points**

3. *Compressed sparse row storage format.* Sparse matrices are stored usually in the so-called Compressed Sparse Row (CSR) format.

   (a) Read the the file csr.pdf (from book by Saad (1996)) about this topic.

   (b) Give two CSR storages of the matrix

$$
\begin{pmatrix}
4 & 0 & 0 & -1 & 0 & 0 & 8 & 10 & 0\\
0 & 10 & -3 & 0 & 0 & 8 & 0 & 0 & 2\\
-1 & 0 & 0 & 0 & 6 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0\\
0 & -6 & 0 & 0 & 0 & 11 & 0 & 0 & 7
\end{pmatrix}.
$$

   2 points

4. *M-matrices.* The class of so-called M-matrices will become important in the lecture.
   A matrix $A \in \mathbb{R}^{n\times n}$ is called M-matrix, if it satisfies the following conditions

   1. $a_{ij} \le 0$ for $i, j = 1, \ldots, n$, $i \ne j$,
   2. A is non-singular and A^{-1} is non-negative, i.e., all entries of A^{-1} are non-negative.

Prove the following statement: Both an M-matrix and its inverse possess positive diagonal entries. **2 points**

The exercise problems should be solved in groups of four students. The solutions have to be submitted until **Monday, Jan. 13, 2025, 10:00 a.m.** via the whiteboard.