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Numerical Mathematics II

Exercise Problems 08

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Matrix exponential.* Show the properties of the matrix exponential given in Lemma 2.25, lecture notes *Numerical Methods for Ordinary Differential Equations.* **6 points**

2. *Stability function at* ∞*.* Consider a Runge–Kutta method $(A, \boldsymbol{b}, \boldsymbol{c})$ with s stages and assume that A is non-singular. Prove the following statement: It holds $R(\infty) = 0$ if one of the following conditions is satisfied:

   (a) $a_{si} = b_i$ for $i = 1, \ldots, s$, or

   (b) $a_{i1} = b_1$ for $i = 1, \ldots, s$, $b_1 \neq 0$.

   Hint: Derive first a formula for $R(\infty)$. **3 points**

3. *Second order boundary value problem with first order term and upwind discretization, GMRES with restart.* In addition to the methods from Exercise 07, Problem 03, the method GMRES(restart) with

   $$\text{restart} \in \{5, 10, 20, 30, 40, 50\}$$

   should be applied for solving the arising linear systems of equations. If you use MATLAB, you can use the build-in routine with the stopping criterion $tol = 10^{-10}$. For other languages, stop the iteration with the same criterion as in Exercise 07, Problem 03. The maximal number of outer iterations should coincide with the dimension of the problem and the initial iterate should be the zero vector. How do the numbers of iterations of GMRES(restart) compare with the other methods from Exercise 07, Problem 03(d) ? Are there trends with respect to the parameter ε or with respect to the parameter restart ? **4 points**

The exercise problems should be solved in groups of four students. The solutions have to be submitted until **Monday, Dec. 09, 2024, 10:00 a.m.** via the whiteboard.