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Numerical Mathematics II

Exercise Problems 02

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Repetition: Analytical solution of an initial value problem.* Solve the following initial value problem

$$xy'(x) + 3y(x) = x^2, \quad x_0 = 0,\ y_0 = 0.$$

4 points

2. *Estimate for a sequence of real numbers.* Assume that for real numbers x_n, $n = 0, 1, \ldots$, the inequality

$$|x_{n+1}| \le (1+\delta)\,|x_n| + \beta$$

holds with constants $\delta > 0$, $\beta \ge 0$. Then, it holds that

$$|x_n| \le e^{n\delta}\,|x_0| + \frac{e^{n\delta}-1}{\delta}\beta, \quad n = 0, 1, \ldots.$$

3 points

3. *Boundary value problem and the convergence of its finite difference approximation.* Consider the boundary value problem

$$\begin{aligned} -u'' &= f \quad \text{in } (0,1), \\ u(0) &= a, \\ u(1) &= b. \end{aligned}$$

(a) Solve this problem analytically for

$$f(x) = -6\pi\cos(3\pi x) + 9\pi^2 x \sin(3\pi x),$$

and $a = b = 0$. **4 points**

(b) Solve this problem numerically using the discretization described in Exercise Sheet 01, Problem 3, with $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$. If you use MATLAB, you can use the backslash command. Give the error of the computed solution u_h to the analytic solution u in the following norm

$$\|u - u_h\|_{l^2} = \left(\frac{1}{N-1}\sum_{i=1}^{N-1}(u(x_i) - u_i)^2\right)^{1/2},$$

where N is the number of nodes. **4 points**

(c) Use the following ansatz of the convergence order

$$\|u - u_h\|_{l^2} = ch^\alpha.$$

Compute α by using the results on the two finest grids. **2 points**

The exercise problems should be solved in groups of four students. The solutions have to be submitted until **Monday, Oct. 28, 2024, 10:00 a.m.** via the whiteboard.