TU Berlin, Scientific Computing, WS17/18
Homework assignment #2

November 21, 2017

Please return this assignment by Friday, Dec. 8, 2017. Please send a zip file by e-mail to juergen.fuhrmann@wias-berlin.de which contains the source code and a pdf describing your answer. Please prefix file names with your last names, e.g. `Müller-Nguyen-HA2.zip`.

1 Problem description

Given:

- Domain $\Omega = (0,1)$
- Right hand side $f : \Omega \to \mathbb{R}$, $f = 1$
- Boundary values $v_L, v_R = 0$
- Transfer coefficient $\alpha = 1$

Search function $u : \Omega \to \mathbb{R}$ such that

$$\begin{aligned} -u'' &= f \quad \text{in } \Omega \\ -u'(0) + \alpha(u(0) - v_L) &= 0 \\ u'(1) + \alpha(u(1) - v_R) &= 0 \end{aligned}$$

2 Tasks

1. Calculate the exact solution of this problem
   - What is the limit of this solution for $\alpha \to \infty$?
2. Implement the finite difference discretization as a linear tridiagonal system on an equidistributed mesh with $N = 2^k + 1$ points with $k = 6 \ldots 14$
   Use the `numcxx` library or another equivalent tool (e.g. Eigen) for this purpose.
   - The library is installed in the UNIX pool and avilable via the course homepage.
   - Hint: have a look at the slides of lecture 05.
3. Use different solution strategies to solve the resulting linear system of equations:
   a) TDMA (Progonka)
   b) Dense matrix direct solver (e.g. LAPACK via numcxx)
   c) Sparse matrix direct solver (e.g. UMFPACK via numcxx)
   d) Simple iterative solver (e.g. Jacobi via numcxx)
   e) Preconditioned CG solver (e.g. via numcxx)
   - Check the results against the exact solution. What happens if N is increased ?
   - Provide timings. Which method is the fastest ?
   - Hint: use e.g. `numcxx::cpu_clock()`
   - What happens for values of the transfer coefficient $\alpha = 1, 10, 100, 1.0 \cdot 10^5, 1.0 \cdot 10^{10}, 1.0 \cdot 10^{20}$?