Scientific Computing 16/17, Homework #2

Please return this assignment by Friday, Nov. 25. Please send a zip or tgz file by e-mail to juergen.fuhrmann@wias-berlin.de which contains the source code and a pdf describing your answer. Please prefix file names with your last names, e.g. `Müller-Nguyen-HA2.tgz`.

1. Problem description

Given:

- Domain $\Omega = (0,1)$
- outer normal $\mathbf{n}$
- Right hand side $f : \Omega \to \mathbb{R}$, $f = 1$
- "Conductivity" $\lambda = 1$
- Boundary value $v : \Gamma \to \mathbb{R}$, $v = 0$
- Transfer coefficient $\alpha = 1$

Search function $u : \Omega \to \mathbb{R}$ such that

$$
\begin{aligned}
-\nabla \cdot \lambda \nabla u &= f \quad \text{in } \Omega \\
-\lambda \nabla u \cdot \mathbf{n} + \alpha(u - v) &= 0 \quad \text{on } \Gamma
\end{aligned}
$$

2. Tasks

1. Calculate the exact solution of this problem
   - What is the limit of this solution for $\alpha \to \infty$?
2. Implement the finite volume discretization as a linear tridiagonal system on an equidistributed mesh with $N = 2^k + 1$ points with $k = 8 \dots 16$ Use the `numcxx` library or another equivalent tool for this purpose.
   - The library is now avilable via the course homepage.
   - Hint: have a look at the slides of lecture 06.
3. Use different solution strategies to solve the resulting linear system of equations:
   a) TDMA (Progonka)
   b) Dense matrix direct solver (e.g. LAPACK via numcxx)
   c) Sparse matrix direct solver (e.g. UMFPACK via numcxx)
   d) Iterative solver (e.g. Jacobi via numcxx)
   - Check the results against the exact solution. What happens if N is increased ?
   - Provide timings. Which method is the fastest ?
   - Hint: use e.g. `numcxx::cpu_clock()`
   - What happens for values of the transfer coefficient $\alpha = 1, 10, 100, 1.0 \cdot 10^5, 1.0 \cdot 10^{10}, 1.0 \cdot 10^{20}$?